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EXISTENCE AND UNIQUENESS OF GLOBAL SOLUTIONS OF THE MODIFIED KS-CGL EQUATIONS FOR FLAMES GOVERNED BY A SEQUENTIAL REACTION*

Boling Guo

(*Institute of Applied Physics and Computational Math., Beijing 100088, PR China*)

Binqiang Xie†

(*The Graduate School of China Academy of Engineering Physics, P.O. Box 2101, Beijing 100088, PR China*)

Abstract

In this paper, we are concerned with the existence and uniqueness of global solutions of the modified KS-CGL equations for flames governed by a sequential reaction, where the term $|P|^2P$ is replaced with the generalized form $|P|^{2\sigma}P$, see [18]. The main novelty compared with [18] in this paper is to control the norms of the first order of the solutions and extend the global well-posedness to three dimensional space.

Keywords existence and uniqueness; modified KS-CGL; global solutions

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1 Introduction

This paper is devoted to the existence and uniqueness of global solutions of the following coupled modified Kuramoto-Sivashinsky-complex Ginzburg-Landau (GKS-CGL) equations for flames

$$\partial_t P = \xi P + (1+i\mu)\Delta P - (1+i\nu)|P|^{2\sigma}P - \nabla P\nabla Q - r_1 P\Delta Q - gr_2 P\Delta^2 Q, \quad (1.1)$$

$$\partial_t Q = -\Delta Q - g\Delta^2 Q + \delta\Delta^3 Q - \frac{1}{2}|\nabla Q|^2 - \eta|P|^2, \quad (1.2)$$

with the periodic initial conditions

$$P(x+Le_i,t) = P(x,t), \quad Q(x+Le_i,t) = Q(x,t), \quad x\in\Omega,\ t\geq 0, \quad (1.3)$$

$$P(x,0) = P_0(x), \quad Q(x,0) = Q_0(x), \quad x\in\Omega, \quad (1.4)$$

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†Corresponding author. E-mail: xbq211@163.com

where Ω is a box with length L denoted T^n $(n = 1, 2, 3)$. The complex function $P(x, t)$ denotes the rescaled amplitude of the flame oscillations, and the real function $Q(x, t)$ is the deformation of the first front. The Landau coefficients μ, ν and the coupling coefficient $\eta > 0$ are real, while r_1 and r_2 are complex parameters of the form $r_1 = r_{1r} + ir_{1i}$, $r_2 = r_{2r} + ir_{2i}$, respectively. The coefficient $g > 0$ is proportional to the supercriticality of the oscillatory mode, $\delta > 0$ is a constant, $L > 0$ is the period and e_i is the standard coordinate vector, and the coefficient $\xi = \pm 1$. The parameter σ, μ, ν satisfy

$$\text{(A1)}\quad 1 < \sigma < \frac{1}{\sqrt{1 + (1 + \frac{\mu - \nu}{2}^2)} - 1},$$

$$\text{(A2)}\quad \sigma \leq \frac{\sqrt{1 + \mu^2}}{4\sqrt{1 + \mu^2} - 4}.$$

For $\delta = 0$, the coupled GKS-CGL equations (1.1) and (1.2) are reduced to the classical KS-CGL equations [1], which describe the nonlinear interaction between the monotonic and oscillatory modes of instability of the two uniformly propagating flame fronts in a sequential reaction. Specifically, they describe both the long-wave evolution of the oscillatory mode near the oscillatory instability threshold and the evolution of the monotonic mode. For the background of the uniformly propagating premixed flame fronts and the derivation of the KS-CGL model, one refer to [1-4] for details. If there exist no coupling with the monotonic model, then equation (1.1) is the well-known CGL equation that describes the weakly nonlinear evolution of a long-scaled instability [5]. For $\delta = 0$ and the coupled coefficient $\eta = 0$ in equation (1.2), equation (1.2) reduces to the well known KS equation [6], which governs the flame front's spatio-temporal evolution and produces monotonic instability. It's seen that the coupled GKS-CGL equations (1.1) and (1.2) can better describe the dynamical behavior for flames governed by a sequential reaction, since they generalize the KS equations, the CGL equations, and the KS-CGL equations.

So far, the mathematical analysis and physical study about the CGL equation and KS equation have been done by many researchers. For example, the existence of global solutions and the attractor for the CGL equation were studied in [7-11]. For some other results, see [12-15] and reference therein. However, little progress has been made for the coupled KS-CGL equations which are derived to describe the nonlinear evolution for flames by A.A. Golovin, et. al. [1], who studied the traveling waves of the coupled equations numerically and the spiral waves in [16], where new types of instabilities are exhibited. Meanwhile, there are few works to consider mathematical analytical properties of the KS-CGL equations and the generalized KS-CGL equations, even the existence and uniqueness of the solutions.