Ann. of Appl. Math.
32:2(2016), 111-132

GLOBAL FINITE ENERGY WEAK SOLUTION TO THE VISCOUS QUANTUM NAVIER-STOKES-LANDAU-LIFSHITZ-MAXWELL MODEL IN 2-DIMENSION*

Boling Guo

(*Institute of Applied Physics and Computational Math., China Academy of Engineering Physics, Beijing 100088, PR China*)

Guangwu Wang†

(*Graduate School of China Academy of Engineering Physics, Beijing 100088, PR China*)

Abstract

In this paper, we prove the global existence of the weak solution to the viscous quantum Navier-Stokes-Landau-Lifshitz-Maxwell equations in two-dimension for large data. The main techniques are the Faedo-Galerkin approximation and weak compactness theory.

Keywords global finite energy weak solution; viscous quantum Navier-Stokes-Landau-Lifshitz-Maxwell system; Faedo-Galerkin method

2000 Mathematics Subject Classification 35A01; 35D30; 35M31; 35Q40; 35Q61

1 Introduction

In studying the dispersive theory of magnetization of ferromagnets, we also consider the viscous quantum of a fluid on motion under the Maxwell electric-magnetic field, that is, the macroscopic motion of a fluid and the quantum effects and the interactions between electrons in microscopic will be considered similarly.

In this paper we study the viscous quantum Navier-Stokes-Landau-Lifshitz-Maxwell system (QNSLLM) in $(0, T) \times \Omega$:

$$\partial_t \rho + \mathrm{div}(\rho u) = \nu \Delta \rho, \tag{1.1}$$

$$\begin{aligned}\partial_t(\rho u) + \mathrm{div}(\rho u \otimes u) + \frac{\nabla P}{m} = &-\frac{\rho e}{m}(E + u \times B) + \frac{\mu \hbar^2}{2m^2}\rho \nabla\Big(\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}\Big) + \nu\Delta(\rho u)\\ &-\frac{\rho u}{\tau} - \lambda \nabla \cdot \Big(\nabla d \odot \nabla d - \frac{|\nabla d|^2}{2} I\Big),\end{aligned} \tag{1.2}$$

*Manuscript received March 3, 2016
†Corresponding author. E-mail: yunxianwgw@163.com

$$d_t + u \cdot \nabla d + \alpha_1 d \times (d \times (\Delta d + B)) = \alpha_2 d \times (\Delta d + B), \tag{1.3}$$
$$E_t - \nabla \times B = e\rho u, \tag{1.4}$$
$$B_t + \nabla \times E = -\lambda m(d_t + u \cdot \nabla d), \tag{1.5}$$
$$\nabla \cdot B = 0, \quad |d(x,t)| = 1. \tag{1.6}$$

Here we denote by $\Omega \subset \mathbf{R}^2$ the two-dimensional 2D-periodic domain, that is, $\Omega = \{x = (x_1, x_2)|\, |x_i| < D; i = 1, 2\}$. ρ and u represent the density and the velocity field of the flow respectively. P is the pressure function. We consider the isentropic case $P = A\rho^\gamma$ ($A > 0$ is a constant). $d : \Omega \to S^1$, the unit sphere in $\mathbf{R}^2$, denotes the magnetization field. E and B represent the electric field and the magnetic field respectively. The physic constants m, e, $\hbar$ are positive and represent the mass, the charge of the particle and Planck constants respectively. ν and μ are positive viscosity constants. The positive constants λ, τ and β represent respectively the competition between kinetic energy and potential energy, the relaxation time of electron and microscopic elastic relaxation time for the molecular orientation field. α_2 is a positive constant, and $\alpha_1 \geq 0$ is Gilbert damping coefficient. $\nabla\cdot$ denotes the divergence operator, and $\nabla d \odot \nabla d$ denotes the 2×2 matrix whose (i, j)-the entry is given by $\nabla_i d \cdot \nabla_j d$ for $1 \leq i, j \leq 2$. The expression $\frac{\Delta\sqrt{\rho}}{\sqrt{\rho}}$ can be interpreted as the quantum Bohm potential.

Roughly speaking, system (1.1)-(1.6) is a coupling between the viscous isentropic quantum Navier-Stokes equations and Landau-Lifshitz-Maxwell equations. This model can be used to describe the dispersive theory of magnetization of ferromagnets with the electromagnetic field.

We call a function $f(x)$ is a $2D$-periodic if $f(x + 2De_i) = f(x)$, $i = 1, 2$, where (e_1, e_2) forms the unit orthogonal basis of $\mathbf{R}^2$, $D > 0$ is a constant.

For system (1.1)-(1.6), we impose the following initial conditions

$$\rho|_{t=0} = \rho_0(x), \quad u|_{t=0} = u_0(x), \quad E|_{t=0} = E_0(x), \quad d|_{t=0} = d_0(x), \quad B|_{t=0} = B_0(x), \tag{1.7}$$

which satisfy that

$$\begin{aligned} &\rho_0(x) > 0, \\ &|d_0(x)| = 1, \quad d_0(x) \in H^2(\Omega), \quad \inf_x d_0^2 > 0, \\ &E_0(x), \quad B_0(x) \in L^2(\Omega). \end{aligned} \tag{1.8}$$

Furthermore, we always assume that $\rho_0(x), u_0(x), d_0(x), E_0(x), B_0(x)$ are 2D-periodic.

Firstly setting $E = B = 0$, d is a constant vector, and using a effective velocity transformation [18] system (1.1)-(1.6) becomes the isentropic compressible quantum Navier-Stokes equation (IQCNS). Set $\mu = 0$, we get the isentropic compressible