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THE ASYMPTOTIC PRESERVING UNIFIED GAS KINETIC SCHEME FOR GRAY RADIATIVE TRANSFER EQUATIONS ON DISTORTED QUADRILATERAL MESHES*†

Wenjun Sun‡, Qinghong Zeng, Shanggui Li

(*Institute of Applied Physics and Computational Math., P.O. Box 8009, Beijing 100088, PR China*)

Abstract

In this paper, we consider the multi-dimensional asymptotic preserving unified gas kinetic scheme for gray radiative transfer equations on distorted quadrilateral meshes. Different from the former scheme [J. Comput. Phys. 285(2015), 265-279] on uniform meshes, in this paper, in order to obtain the boundary fluxes based on the framework of unified gas kinetic scheme (UGKS), we use the real multi-dimensional reconstruction for the initial data and the macro-terms in the equation of the gray transfer equations. We can prove that the scheme is asymptotic preserving, and especially for the distorted quadrilateral meshes, a nine-point scheme [SIAM J. SCI. COMPUT. 30(2008), 1341-1361] for the diffusion limit equations is obtained, which is naturally reduced to standard five-point scheme for the orthogonal meshes. The numerical examples on distorted meshes are included to validate the current approach.

Keywords gray radiative transfer equations; distorted quadrilateral meshes; asymptotic preserving; unified gas kinetic scheme; nine-point diffusion scheme

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1 Introduction

This paper is about the development of a multi-dimensional unified gas kinetic scheme for the numerical solution of the time-dependent gray radiative transfer

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‡Corresponding author. E-mail: sun_wenjun@iapcm.ac.cn

equations on distorted quadrilateral meshes. It is well-known that the gray radiative transfer equations are the modeling in the kinetic scale, where the dynamics of photon transport and collision with material is taken into account. The wide applications of this system include astrophysics, inertial confinement fusion, high temperature flow systems, and many others. Due to the importance and complexity of the system, its study attracts much attention from national laboratories and academic institutes.

Since the gray radiative transfer equations model the radiation energy transport and the energy exchange with the background material, the properties of the background material influence greatly on the behavior of radiation transfer. If the background material opacity is low, the interaction between the radiation and material is weak, and the radiation propagates in a transparent way. On the other hand, if the background material opacity is high, there is severe interaction between radiation and material with a diminishing photon mean free path. As a result, the diffusive radiative behavior will emerge. Then in some sense, this gray radiative transfer equation is a multi-scale problem.

Physically, in order to solve the kinetic scale based radiative transfer equations numerically, the spatial mesh size should be comparable to the photon's mean-free path, which is very small in the optical thick regime. Consequently, the simulation in this regime is associated with huge computational cost. To remedy these difficulties, one of the approaches is to develop the so-called asymptotic preserving (AP) scheme for the kinetic equation. For the AP scheme, as holding the mesh size and time step fixed and letting the Knudsen number goes to zero, the discrete diffusion limit solution will be automatically recovered. The AP schemes were first studied in the numerical solution of steady neutron transport problems by Larsen, Morel and Miller [10], Larsen and Morel [9], and then by Jin and Levermore [5,6]. For unsteady problems, the AP schemes were constructed based on a decomposition of the distribution function between an equilibrium part and its non-equilibrium derivation, see Klar [8], and Jin, Pareschi and Toscani [7] for details. Very recently, based on the unified gas kinetic scheme (UGKS) framework [15], a rather different approach was proposed by Mieussens for a linear radiation transport model [13], where this linear model is just a scalar equation, which does not couple with the material temperature's equation. And then by introducing an angle-moment macro-equation for gray radiation transfer equation, on uniform orthogonal quadrilateral meshes, an asymptotic preserving unified gas kinetic scheme (AP-UGKS) has been developed for the grey and multi-group radiation transfer equations which are composed of radiation transport and material energy equation, see [1, 16] for details.

From the numerical results of UGKS method, we can see that this scheme is