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ASYMPTOTIC BEHAVIOR FOR GENERALIZED GINZBURG-LANDAU POPULATION EQUATION WITH STOCHASTIC PERTURBATION*†

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Abstract

In this paper, we are devoted to the asymptotic behavior for a nonlinear parabolic type equation of higher order with additive white noise. We focus on the Ginzburg-Landau population equation perturbed with additive noise. Firstly, we show that the stochastic Ginzburg-Landau equation with additive noise can be recast as a random dynamical system. And then, it is proved that under some growth conditions on the nonlinear term, this stochastic equation has a compact random attractor, which has a finite Hausdorff dimension.

Keywords Ginzburg-Landau model; additive white noise; random attractor; Hausdorff dimension

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1 Introduction

One of the most important problems in the fields of differential equations is that of the asymptotic behavior of evolution equations. During the last decades, finite-dimension attractors for deterministic systems have been quite well investigated. Particularly in [8,9], the authors were devoted to the global attractors for non-invertible planar piecewise isometric maps and a class of nonhyperbolic piecewise affine maps. They obtained sufficient and necessary conditions for a compact set K to be the global attractor. Recently, Crauel and Flandoli [3] generalized the theory of deterministic attractors to the stochastic case. However, due to the introduction of random influences, the system are pushed out every bounded set with probability one. Therefore, we need to define the random attractor for the stochastic system. As far as we know, there are several different definitions, see [1,3,12,13]. In [12,13],

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the authors considered the attractors for the Markov semigroup generated by a stochastic differential equation. While, in [1], they took the attractors as the Ω-limit set for $t \to \infty$ of the trajectories. In this paper, we consider the attractors as a subset of the phase space (as in [3]), which is the Ω-limit set at time $t = 0$ of the trajectories "starting in bounded sets at time $t = -\infty$." For more detailed information about stochastic equations, one can refer to [5].

As we know, the famous Ginzburg-Landau model

$$u_t = -a_1 \nabla^4 u + a_2 \nabla^2 u + a \nabla^2 u^3 + G(u)$$

was proposed in [4] for growth and dispersal in population. After that, there are a series of research on the existence, uniqueness and regularity of its global solutions, see [2,11]. In this paper, we focus on the asymptotic behavior of the following nonlinear parabolic type equation of higher order perturbed by additive white noise

$$\begin{cases} u_t = -a_1 \nabla^4 u + a_2 \nabla^2 u + \nabla^2 g(u) + G(u) + \sum_{j=1}^{m} \phi_j \mathrm{d} w_j(t), & (x,t) \in D \times (0,T), \\ \dfrac{\partial u}{\partial \nu} = 0, \quad \dfrac{\partial \nabla^2 u}{\partial \nu} = 0, & (x,t) \in \partial D \times [0,T], \\ u(x,0) = u_0(x), \quad x \in \overline{D}, \end{cases} \tag{1}$$

where $a_1 > 0,\ a_2 > 0,\ D \subset R^n$ is a bounded open set with regular boundary ∂D, ν is the outward normal vector of the boundary ∂D, $\phi_j \in D(A)$ with $j = 1, \cdots, m$ being time independent defined on D, and $\{\omega_j\}_{j=1}^m$ are independent two-sided real-valued Wiener processes on a complete probability space $(\Omega, \mathcal{F}, P)$, $\frac{\partial \phi_j}{\partial t} = 0,\ j = 1, 2, \cdots, m$.

Introduce $H = L^2(D),\ V = H_0^1(D)$, and define $Au = -\triangle u,\ D(A) = \{u \in H^2(D) :\ u = 0 \text{ on } \partial D\} = V \cap H^2(D)$, then the operator A is positive, linear and self-adjoint, which has a sequence of eigenvalues $0 < \lambda_1 < \lambda_2, \cdots,\ \lambda_j \to \infty$ and a family of elements ω_j of $D(A)$ which is orthonormal in H such that $A\omega_j = \lambda_j \omega_j$, for any j. For the first eigenvalue λ_1, we have $||x||^2 \geq \lambda_1 |x|^2,\ |\triangle x|_{L^2}^2 \geq \lambda_1 ||x||^2$ for all $x \in D(A)$. Denote by $(\cdot,\cdot)$ and $|\cdot|$ the inner product and norm in H respectively, and by $((\cdot,\cdot))$ and $||\cdot||$ the inner product and norm in $V = H_0^1(D)$ respectively.

We assume that:

(H_1) $0 < g'(u) < \gamma,\ |g''(u)| < \beta$,

(H_2) $G(u) = \sum_{k=0}^{2p-1} d_k u_k$ with $d_{2p-1} < 0$.

We present the following theorem of [3] for the existence of global attractor.

Theorem 1.1 *Suppose that φ is an RDS on a Polish space X, and that there exists a compact $\omega \to K(\omega)$ absorbing every bounded nonrandom set $B \subset X$. Then*