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TRAVELING WAVE SOLUTIONS AND THEIR STABILITY OF NONLINEAR SCHRÖDINGER EQUATION WITH WEAK DISSIPATION*†

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Abstract

In this paper, several new constant-amplitude and variable-amplitude wave solutions (namely, traveling wave solutions) of a generalized nonlinear Schrödinger equation are investigated by using the extended homogeneous balance method, where the balance method is applied to solve the Riccati equation and the reduced nonlinear ordinary differential equation, respectively. In addition, stability analysis of those solutions are also conducted by regular phase plane technique.

Keywords nonlinear Schrödinger equation; extended homogeneous balance method; amplitude wave solutions; stability

2000 Mathematics Subject Classification 35B40; 35K58; 35B32

1 Introduction

The investigation of temporal or spatial dynamics for nonlinear Schrödinger equations is an important and interesting subject, see, for example, [1-11,13-18,20] for details. In particular, there are many papers which have paid more attention to the dynamics of the following (2+1)-dimensional cubic nonlinear Schrödinger (NLS) equation without dissipation

$$iu_t + \alpha u_{xx} + \beta u_{yy} + \gamma |u|^2 u = 0, \tag{1.1}$$

where $u = u(x, y, t)$ is a complex-valued function, α, β, and γ are real constants, and subscripts represent partial derivatives. As we know, the NLS equation is referred to as an approximate model of the evolution of a nearly monochromatic wave of small amplitude of pulse propagation in Langmuir waves in a plasma, optical fibers and gravity waves on deep water with different values of parameters.

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Recently, modulational instability of many extended versions of the NLS equation with different dissipations have been investigated in [14,15,17]. In particular, there is a nonlinear dissipative Schrödinger (DissNLS) equation as follows:

$$iu_t + (\alpha - ia)u_{xx} + (\beta - ib)u_{yy} + (\gamma + ic)|u|^2 u + idu = 0, \tag{1.2}$$

where $u = u(x, y, t)$ $(x, y \in R)$ is a complex-valued function, $\alpha, \beta, \gamma, a, b, c$ and d are real constants with a, b and c being all nonnegative, d represents dissipation. Note that, this equation is regarded as a model of weakly nonlinear surface wave, and it can also be regarded as a generalized version of the complex Ginzburg-Landau equation.

Actually, modulational instability corresponds to temporal stability. However, the investigation of traveling wave solutions also plays an important role in the dynamics of nonlinear physical phenomena, see, for example, Zhang et al. [16], Feng and Meng [19], Nguyen [21]. To my best, except for particular parameters, there are no exact analytical solutions of (2+1)-dimensional NLS equation, so sometimes one has to resort to computer numerical simulations in order to investigate the dynamics of NLS, thus it is necessary to obtain exact solutions by certain analytic technique. Therefore, in order to better understand the dynamical behavior of the dissipative nonlinear Schrödinger equation (1.2), in this paper, we will focus on its exact traveling wave solutions and their spatial stability.

The rest of this paper is outlined as follows. Section 2 contains two kinds of exact amplitude traveling wave solutions obtained by the homogeneous balance method. In Section 3, we study the stability of traveling wave solutions of NLS equation by using the regular phase plane method.

Now, we introduce the homogenous balance method and use it to look for special exact solutions of some nonlinear equations. Consider a general partial differential equation

$$H(u, u_x, u_t, u_{xx}, u_{yy}, \cdots) = 0, \tag{1.3}$$

where H is a polynomial function of its arguments, subscripts denote the partial derivatives. We will solve (1.3) by the homogeneous balance method with the following four steps:

<u>Step 1</u> Firstly, take

$$u = f^{(m+n)}(\varphi_x)^m(\varphi_t)^n + \sum_{i+j=0}^{m+n-1} A_{ij} f^{(i+j)}(\varphi_x)^i(\varphi_t)^j, \tag{1.4}$$

where m and n are nonnegative integers, the functions $f = f(\varphi)$ and $\varphi = \varphi(x, t)$, and the coefficients A_{ij} are all to be determined. Substituting (1.4) into (1.3), the integers m and n will be determined.