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A LOTKA-VOLTERRA PREY-PREDATOR SYSTEM WITH FEEDBACK CONTROL EFFECT*

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Abstract

In this paper, we propose a Lotka-Volterra prey-predator system with discrete delays and feedback control. Firstly, we show that solution of the system is bounded. Secondly, we obtain sufficient condition for the global stability of the unique positive equilibrium to the system.

Keywords Lotka-Volterra prey-predator system; feedback control; global stability

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1 Introduction

During the last decades, many scholars have investigated the dynamic behaviors of systems with feedback control (see [1-10]). Gopalsamy and Weng [1] first introduced a feedback control variable into a competitive system:

$$\begin{aligned} \frac{\mathrm{d}n(t)}{\mathrm{d}t} &= rn(t)\Big[1-\Big(\frac{a_1n(t)+a_2n(t-\tau)}{K}-cu(t)\Big)\Big], \\ \frac{\mathrm{d}u(t)}{\mathrm{d}t} &= -au(t)+bn(t-\tau). \end{aligned} \tag{1.1}$$

They investigated the stability property of the positive equilibrium. Li and He [2] investigated the following single-species food-limited system with feedback control and delay

$$\begin{aligned} \frac{\mathrm{d}u(t)}{\mathrm{d}t} &= u(t)\Big(\frac{r(K-u(t))}{K+au(t)}-cv(t-\tau)\Big), \\ \frac{\mathrm{d}v(t)}{\mathrm{d}t} &= -dv(t)+bu(t-\tau). \end{aligned} \tag{1.2}$$

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Shi, Chen and Wang [3] discussed a Lotka-Volterra prey-predator system with feedback control

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)[r_1 - a_{11}x_1(t) - a_{12}x_2(t-\tau_1)],\\
\dot{x}_2(t) &= x_2(t)[r_2 + a_{21}x_1(t-\tau_2) - a_{22}x_2(t) - cu(t)],\\
\dot{u}(t) &= -eu(t) + dx_2(t).
\end{aligned} \tag{1.3}$$

They showed that a suitable feedback control on predator species can make the dynamic of prey species changes from extinction to global stability.

Note that all the above papers assumed that the feedback control item $u(t)$ is influenced by the population level of the matured species. It is naturally to ask: What would happen if the feedback control item $u(t)$ is influenced by the population level of the immature species.

Motivated by the above works, in this paper, we consider:

$$\begin{aligned}
\dot{x}_1(t) &= x_1(t)[r_1 - a_{11}x_1(t) - a_{12}x_2(t-\tau_1) - cu(t)],\\
\dot{x}_2(t) &= x_2(t)[r_2 + a_{21}x_1(t-\tau_2) - a_{22}x_2(t)],\\
\dot{u}(t) &= -eu(t) + dx_1(t),
\end{aligned} \tag{1.4}$$

where $u(t)$ is the indirect control variable, $r_i > 0$, $\tau_i > 0$, $a_{ij} > 0$, $i, j = 1, 2$, $c > 0$, $e > 0$, $d > 0$, $\tau = \max\{\tau_1, \tau_2\}$, subject to

$$x_i(\theta) = \phi_i(\theta), \quad u(\theta) = \psi(\theta), \quad \theta \in [-\tau, 0], \tag{1.5}$$

$\phi_i(\theta)$ and $\psi(\theta)$ are nonnegative and bounded continuous functions on $[-\tau, 0]$.

The organization of the paper is as follows. In the next section, we investigate the existence of positive equilibrium and the boundness property of the system; In Section 3, by constructing a Lyapunov functional, we show the global stability of the equilibriums under some assumptions.

2 Preliminaries

System (1.4) always admits a boundary equilibrium $O(0, 0, 0)$.

Suppose that assumption $r_1a_{22} - a_{12}r_2 > 0$ holds. It is not difficult to verify that system (1.4) has a unique positive equilibrium (x_1^*, x_2^*, u^*):

$$x_1^* = \frac{(r_1a_{22} - a_{12}r_2)(a_{11}e + c_1d_1)}{ea_{21}a_{12} + a_{22}(a_{11}e + c_1d_1)}, \quad x_2^* = \frac{r_2(a_{11}e - c_1d_1) + ea_{21}r_1}{ea_{21}a_{12} + a_{22}(a_{11}e + c_1d_1)}, \quad u^* = \frac{d_1}{e}x_1^*. \tag{2.1}$$

Lemma 2.1 *Let* $(x_1(t), x_2(t), u(t))^{\mathrm{T}}$ *be a solution of system* (1.4) *with the initial condition* (1.5)*, Then* $(x_1(t), x_2(t), u(t))$ *is positive and bounded for all* $t > 0$.

Proof Obviously, solutions $(x_1(t), x_2(t), u(t))^{\mathrm{T}}$ of system (1.4) with the initial condition (1.5) are positive for all $t > 0$. By the first equation of system (1.4), we have