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ASYMPTOTIC NORMALITY OF THE NONPARAMETRIC KERNEL ESTIMATION OF THE CONDITIONAL HAZARD FUNCTION FOR LEFT-TRUNCATED AND DEPENDENT DATA*†

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Abstract

Under some mild conditions, we derive the asymptotic normality of the Nadaraya-Watson and local linear estimators of the conditional hazard function for left-truncated and dependent data. The estimators were proposed by Liang and Ould-Saïd [1]. The results confirm the guess in Liang and Ould-Saïd [1].

Keywords asymptotic normality; Nadaraya-Watson estimation; local linear estimation; conditional hazard function; left-truncated data

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1 Introduction

Left-truncated data often occurs in astronomy, economics, epidemiology and biometry; see, e.g., Woodroofe [2], Feigelson and Babu [3], Wang et al. [4], He and Yang [5]. Since Ould-Saïd and Lemdani [6] proposed a new nonparametric estimator (of NW type) of the nonparametric regression function under a left-truncation model, the nonparametric statistical inference for left-truncated data has been received an increasing interest in the literatures. For the case of estimating the conditional mean function, see Liang et al. [7,8], Liang [9], Wang [10]; for the case of estimating the conditional quantile function, see Lemdani et al. [11], Liang and de Uña-Álvarez [12,13], Wang [14]; for the case of estimating the conditional density function, see Ould-Saïd and Tatachak [15], Liang [16], Liang and Baek [17].

Recently, Liang and Ould-Saïd [1] proposed a plug-in weighted estimator of the

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conditional hazard rate for left-truncated and dependent data. They obtained asymptotic normality of the estimator and compared the finite sample performance with the Nadaraya-Watson (NW) and local linear (LL) estimators proposed by them, but they did not give the asymptotic normality of the NW and LL estimators. This paper will make up for this problem.

The rest of this paper is organized as follows. In Section 2, we first introduce the left-truncation model, some results, and the NW and LL estimators of the conditional hazard function in the left-truncation model. The asymptotic normality of the NW and LL estimators are stated in Section 3, while their proofs are given in Section 4.

2 Estimators

2.1 Preliminary

Let (Y, T) be random variables, where Y is the variable of interest (regarded as the lifetime in survival analysis) with a distributed function (df) $\widetilde{F}(\cdot)$, and T is the random left-truncation variable with a continuous d.f. $G(\cdot)$. In the random left-truncation model, one can observe (Y, T) when $Y \geq T$, whereas nothing is observed when $Y < T$. Let $\theta = \mathbb{P}\ (Y \geq T)$, then θ is the probability that Y can be observed. It is obvious we need to assume that $\theta > 0$. Suppose that X is the associate covariate with a density function $l(\cdot)$, and (X, Y) is independent of T. Let $f(\cdot|x)$ and $S(\cdot|x)$ denote the conditional density function and the conditional survival function of Y given $X = x$, respectively. Then for all $y \in \mathbb{R}$ and $S(y|x) > 0$, the corresponding conditional hazard function $\lambda(y|x)$ equals

$$\lambda(y|x) = \frac{f(y|x)}{S(y|x)}. \tag{2.1}$$

Let $\{(X_i, Y_i, T_i), 1 \leq i \leq N\}$ be a sequence of random vectors from (X, Y, T). As a result of left-truncation, n-the size of the actually observed sample, is random with $n \leq N$ and N being unknown. Since N is unknown and n is known (although random), the results are not stated with respect to the probability measure $\mathbb{P}$ (related to the N-sample) but involve the conditional probability P with respect to the actually observed n-sample. Also $\mathbb{E}$ and E denote the expectation operators under $\mathbb{P}$ and P, respectively.

In the sequel, the observed sample $\{(X_i, Y_i, T_i), 1 \leq i \leq n\}$ is assumed to be a stationary α-mixing sequence. Recall that a stationary process $\{U_i, i \geq 1\}$ is called α-mixing or strongly mixing, if the α-mixing coefficient

$$\alpha(n) := \sup_{k\geq 1} \sup_{A\in\mathcal{A}_1^k, B\in\mathcal{A}_{k+n}^{+\infty}} |P(AB) - P(A)P(B)|$$