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THE STABILITY OF A REACTION-DIFFUSION LOTKA-VOLTERRA COMPETITIVE SYSTEM WITH NONLOCAL DELAYS AND FEEDBACK CONTROLS*†

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Abstract

In this paper, we consider a Lotka-Volterra competitive system with nonlocal delays and feedback controls. Using the Lyapunov functional and iterative technique method, we investigate the global stability and extinction of the system. Also, we show the influence of feedback controls on dynamic behaviors of the system. Some examples are presented to verify our main results.

Keywords extinction; feedback control; reaction-diffusion; Lotka-Volterra; nonlocal delays

2000 Mathematics Subject Classification 35K57; 35B35; 35B40; 35Q92

1 Introduction and Main Results

In this paper, we consider the following reaction-diffusion Lotka-Volterra competitive system with nonlocal delays and feedback controls

$$\begin{aligned}
&\frac{\partial u_1}{\partial t}-D_1\Delta u_1=u_1\left(b_1-a_{11}u_1-a_{12}\int_{-\infty}^{t}\int_0^{\pi}G_1(x,y,t-s)f_1(t-s)u_2(s,y)\mathrm{d}y\mathrm{d}s-c_1v_1\right),\\
&\frac{\partial u_2}{\partial t}-D_2\Delta u_2=u_2\left(b_2-a_{21}\int_{-\infty}^{t}\int_0^{\pi}G_2(x,y,t-s)f_2(t-s)u_1(s,y)\mathrm{d}y\mathrm{d}s-a_{22}u_2-c_2v_2\right),\\
&\frac{\partial v_1}{\partial t}-D_3\Delta v_1=-e_1v_1+d_1u_1,\\
&\frac{\partial v_2}{\partial t}-D_4\Delta v_2=-e_2v_2+d_2u_2,
\end{aligned}\tag{1.1}$$

for $t>0,\ x\in(0,\pi)$, under the homogeneous Neumann boundary conditions

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$$\frac{\partial u_i}{\partial x} = \frac{\partial v_i}{\partial x} = 0, \quad t > 0,\ x = 0, \pi,\ i = 1, 2, \tag{1.2}$$

and initial conditions

$$\begin{aligned} &u_i(\theta, x) = \phi_i(\theta, x) \geq 0, \quad (\theta, x) \in (-\infty, 0] \times [0, \pi], \\ &v_i(0, x) = \psi_i(0, x) \geq 0, \quad x \in (0, \pi),\ i = 1, 2. \end{aligned} \tag{1.3}$$

In system (1.1), u_i denotes the population density of the i-th species; v_i denotes the feedback control variable; b_1 and b_2 are the intrinsic growth rates; a_{11} and a_{22} are the rates of the intra-specific competition of the first and second species respectively; a_{12} and a_{21} are the rates of the inter-specific competition of the first and second species respectively; c_i, e_i, d_i are coefficients of the feedback control variable; D_i is the diffusion rate. All the parameters in system (1.1) are positive constants. The boundary conditions (1.2) imply that the populations and feedback control variable do not move across the boundary $x = 0, \pi$. We assume that the kernel $G_i(x, y, t)f_i(t)$ depends on both the spatial and the temporal variables. The delay in this type of model formulation is called a spatio-temporal delay or nonlocal delay (as we shall show below how G_i are chosen).

The following two-species autonomous competitive system

$$\begin{aligned} x_1'(t) &= x_1(t)(b_1 - a_{11}x_1(t) - a_{12}x_2(t)), \\ x_2'(t) &= x_2(t)(b_2 - a_{21}x_1(t) - a_{22}x_2(t)), \end{aligned} \tag{1.4}$$

where $b_i, a_{ij},\ i, j = 1, 2$ are positive constants, has been discussed in many books on mathematical ecology (for example [1]). If the coefficients of system (1.4) satisfy $\frac{a_{11}}{a_{21}} > \frac{b_1}{b_2} > \frac{a_{12}}{a_{22}}$, then system (1.4) has a unique positive equilibrium $(\overline{x}_1, \overline{x}_2)$ which is globally attractive, that is, all positive solutions of system (1.4) satisfy $\lim\limits_{t\to+\infty}(x_1(t), x_2(t)) = (\overline{x}_1, \overline{x}_2)$. If the coefficients of system (1.4) satisfy $\frac{b_1}{b_2} > \frac{a_{11}}{a_{21}},\ \frac{b_1}{b_2} > \frac{a_{12}}{a_{22}}$, then system (1.4) is extinct, that is, all positive solutions of system (1.4) satisfy $\lim\limits_{t\to+\infty}(x_1(t), x_2(t)) = (\frac{b_1}{a_{11}}, 0)$.

In [2], the authors argued that in some situation, the equilibrium is not the desirable one (or affordable) and a smaller value is required, which can be explained logically especially in a food limited environment since the circumstance can only withstand a certain amount of populations. Thus we must alter the system structurally by introducing a feedback control variable (Aizerman and Gantmacher [3] or Lefschetz [4]). On the other hand, ecosystem in the real world are continuously disturbed by unpredictable forces which can result in some changes of the biological parameters such as survival rates. We call the disturbance functions to be control variables. Gopalsamy and Weng [5] introduced a feedback control variable into a two species competitive system and discussed the existence of the globally attractive