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SOME PROBLEMS IN RADIATION TRANSPORT FLUID MECHANICS AND QUANTUM FLUID MECHANICS*

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Abstract

We introduce the radiation transport equations, the radiation fluid mechanics equations and the fluid mechanics equations with quantum effects. We obtain the unique global weak solution for the radiation transport fluid mechanics equations under certain initial and boundary values. In addition, we also obtain the periodic region problem of the compressible N-S equation with quantum effect has weak solutions under some conditions.

Keywords radiation transport equation; radiation fluid mechanics equations; fluid mechanics equations with quantum effects

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1 Radiation Transport Equation and Radiation Fluid Mechanics Equations

A radiation transport equation is as follows

$$\frac{1}{c}\frac{\partial I(\upsilon,\Omega)}{\partial t}+\Omega\cdot\nabla I(\upsilon,\Omega)=S(\upsilon)-\sigma_a(\upsilon)I(\upsilon,\Omega)+ \int_0^\infty \mathrm{d}\upsilon'\int_{S^{n-1}}\Big[\frac{\upsilon}{\upsilon'}\sigma_s(\upsilon'\to\upsilon,\Omega'\cdot\Omega)I(\upsilon',\Omega')-\sigma_s(\upsilon\to\upsilon',\Omega\cdot\Omega')I(\upsilon,\Omega)\Big]\mathrm{d}\Omega', \quad (1.1)$$

where $I(\upsilon,\Omega)=I(x,t,\upsilon,\Omega)$ is the radiation intensity, $S(\upsilon)$ is the production rate of photons, $\sigma_a(\upsilon)$ is the absorption rate, $\sigma_s(\upsilon)$ is the scattering rate. In generally, $\sigma_a=O(\rho^\alpha\theta^{-\beta})$, $\alpha>0$, $\beta>0$, where ρ is the density of matter, θ is the temperature of matter, and the radiation intensity of scattering out is

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$$\int_0^\infty \mathrm{d}\upsilon' \int_{S^{n-1}} \sigma_s(\upsilon \to \upsilon', \Omega\cdot\Omega') I(\upsilon,\Omega)\mathrm{d}\Omega',$$

the radiation intensity of scattering in is

$$\int_0^\infty \mathrm{d}\upsilon' \int_{S^{n-1}} \sigma_s(\upsilon' \to \upsilon, \Omega'\cdot\Omega) I(\upsilon',\Omega')\mathrm{d}\Omega',$$

where S^{n-1} is the unit sphere in R^{n-1}.

Define the absorption coefficient and compton scattering nucleus

$$\sigma_a(\upsilon) = c_1\rho\theta^{-\frac{1}{2}} \exp\Big[-\frac{c_2}{\theta^{\frac{1}{2}}}\Big(\frac{\upsilon-\upsilon_0}{\upsilon_0}\Big)^2\Big],$$

$$\sigma_s(\upsilon\to\upsilon',\xi) = \frac{c_3\rho(1+\xi^2)}{[1+\gamma(1-\xi)]^2}\times\Big\{1+\frac{\gamma^{2(1-\xi)^2}}{(1+\xi^2)[1+\gamma(1-\xi)]}\Big\}\times\delta\Big(\upsilon'-\frac{\upsilon}{1+\gamma(1-\xi)}\Big),$$

where $\gamma = c_4\upsilon,\ \xi = \Omega\cdot\Omega',\ c_i\,(i=1,\cdots,4)$ are positive constants, υ_0 is the frequency. We now define the radiation energy density, radiation flux and radiation pressure as follows, respectively

$$\begin{cases} E_r = \dfrac{1}{c}\displaystyle\int_0^\infty \mathrm{d}\upsilon \int_{S^{n-1}} I(\upsilon,\Omega)\mathrm{d}\Omega, \\ F_r = \displaystyle\int_0^\infty \mathrm{d}\upsilon \int_{S^{n-1}} \Omega I(\upsilon,\Omega)\mathrm{d}\Omega, \\ P_r = \dfrac{1}{c}\displaystyle\int_0^\infty \mathrm{d}\upsilon \int_{S^{n-1}} \Omega\otimes\Omega I(\upsilon,\Omega)\mathrm{d}\Omega. \end{cases} \tag{1.2}$$

The radiation transport fluid mechanics equations are

$$\begin{cases} \rho_t + \mathrm{div}(\rho u) = 0, \\ \Big(\rho u + \dfrac{1}{c^2 F_r}\Big)_t + \nabla(\rho u\otimes u + P_m + P_r) = \mathrm{div}S, \\ \Big(\dfrac{1}{2}\rho u^2 + E_m + E_r\Big)_t + \nabla\Big[\Big(\dfrac{1}{2}\rho u^2 + E_m + P_m\Big)u + F_r\Big] = \mathrm{div}(Su + k\nabla\theta), \end{cases} \tag{1.3}$$

where ρ, u, $P_m = P_m(\rho,\theta)$, $E_m = E_m(\rho,\theta)$ and θ are the density, speed, pressure, internal energy and temperature of fluid, respectively. $k = k(\rho,\theta)$ is the thermal conductivity of fluid, S is the viscous tensor

$$S = \lambda\mathrm{div}I + \mu(\nabla u + (\nabla u)^{\mathrm{T}}),$$

where λ and μ are viscous coefficients with $2\lambda+\mu > 0$.

The radiant transport equation through the absorption of photons and after scattering interaction is