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THE QUALITATIVE ANALYSIS OF A TWO SPECIES AMENSALISM MODEL WITH NON-MONOTONIC FUNCTIONAL RESPONSE AND ALLEE EFFECT ON SECOND SPECIES*†

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Abstract

In this paper, we present a two species amensalism model with non-monotonic functional response and Allee effect on second species. Local and global stability of the boundary and interior equilibrium are investigated. By introducing the Allee effect, we show that the boundary equilibrium have changed from unstable node and saddle into saddle-node. Also, the system subject to an Allee effect has increased the time of reach to its stable steady-state solution, but has no influence on the final density of the two species. Our results are supported by numeric simulations.

Keywords amensalism model; Allee effect; non-monotonic functional response; global stability

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1 Introduction

Amensalism is a basic interaction where a species inflicts harm to the other species without any costs or benefits received by the other. For example, the natural jump of locusts has severely reduced the quality of the caterpillar's survival, see [1] and the references cited therein.

In 2003, Sun [2] proposed the following two populations amensalism model:

$$\begin{aligned}\frac{\mathrm{d}x}{\mathrm{d}t}&=r_1x\Big(\frac{k_1-x-\alpha y}{k_1}\Big),\\ \frac{\mathrm{d}y}{\mathrm{d}t}&=r_2y\Big(\frac{k_2-y}{k_2}\Big),\end{aligned}\tag{1.1}$$

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where r_1, r_2, k_1, k_2 and α are all positive constants. The author investigated the stability property of all the possible equilibria of the system.

During the last decade, based on (1.1), many scholars [3-11] have investigated the dynamic behaviors of varies amensalism model and obtained many interesting results, which have greatly enriched and improved this direction. For example: For an amensalism system with strong generic delay kernel, the local asymptotic stability and the existence of Hopf bifurcation of the positive equilibrium were investigated in [4]; sufficient conditions were obtained for the existence of positive periodic solution of a discrete amensalism model with Holling II functional response in [8]; a two species amensalism model with a cover for the first species was investigated in [10], etc.

In particular, stimulated by the functional response of the predator-prey model, Wu [11] argued that it is necessary to use suitable functional response to discrete the relationship between the two species. Generally speaking, there is a saturating state between two species, so it is more practical to describe the saturation phenomenon using the non-monotonic type functional response [12, 13]. So the author proposed the following two species amensalism symbiosis model with non-monotonic functional response:

$$\begin{aligned} \frac{\mathrm{d}x}{\mathrm{d}t} &= x\Big(a_1 - b_1 x - \frac{c_1 y}{d_1 + y^2}\Big), \\ \frac{\mathrm{d}y}{\mathrm{d}t} &= y(a_2 - b_2 y), \end{aligned} \tag{1.2}$$

where a_1, b_1, c_1, d_1, a_2, b_2 are all positive constants. The second species to the first one obey the non-monotonic functional response instead of linearize response. We notice that, adding this functional response in system (1.2) does not change any properties of the equilibrium, but only changes the stability conditions of the boundary equilibrium and the positive equilibrium.

It also brings to our attention that, to this day, still no scholars consider the influence of Allee effect to the amensalism model. Allee effect occurs when species depend upon predator saturation, evading natural enemies, cooperative predation or resource defense, selecting mates and increasing availability of mates [14-16], etc. At lower population densities, any of these factors can dominate. At this time, the Allee effect has great influence on the population. When the Allee effect is sufficiently strong, there is a critical threshold below which population behavior and dynamics may experience change. Consequently, the importance of the Allee effect has been widely recognized in many biological disciplines.

In recent years, there are many papers concerned with the Allee effects of the predator-prey system and competition system, see [17-24] and the references cit-