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GLOBAL EXISTENCE OF MILD SOLUTIONS FOR THE ELASTIC SYSTEM WITH STRUCTURAL DAMPING*†

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Abstract

In this paper, we study the global existence of mild solutions for the semilinear initial-value problems of second order evolution equations, which can model an elastic system with structural damping. This discussion is based on the operator semigroups theory and the Leray-Schauder fixed point theorem. In addition, an example is presented to illustrate our theoretical result.

Keywords C_0-semigroup; elastic systems; structural damping; mild solution; fixed point

2000 Mathematics Subject Classification 35G10; 47N20

1 Introduction

In this paper, we will investigate the following semilinear elastic systems with structural damping

$$\begin{cases} \ddot{u}(t) + \rho\mathscr{A}\dot{u}(t) + \mathscr{A}^2 u(t) = f(t, u(t)), \quad 0 \le t \le b, \\ u(0) = x_0, \quad \dot{u}(0) = y_0, \end{cases} \tag{1.1}$$

where " $\cdot$ " means $\frac{\mathrm{d}}{\mathrm{d}t}$, $\rho \ge 2$ is damping coefficient, $-\mathscr{A}$ is the infinitesimal generator of a C_0-semigroup $T(t)$ $(t \ge 0)$ in Banach space $\mathbb{X}$, and $f : [0, b] \times \mathbb{X} \to \mathbb{X}$ is a given function, $b > 0$ is a constant, $x_0 \in \mathscr{D}(\mathscr{A})$, $y_0 \in \mathbb{X}$.

The consideration of this problem seems to have been initiated by Chen and Russell [1] in 1982. They studied the following linear second order evolution equation

$$\ddot{u}(t) + B\dot{u}(t) + Au(t) = 0 \tag{1.2}$$

in a Hilbert space H with inner product $(\cdot, \cdot)$, where A (the elastic operator) is a positive definite, self-adjoint operator in H, and B (the damping operator) is a positive self-adjoint operator in H. They reduced (1.2) to the first order equation in $H \times H$,

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$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} A^{1/2}u \\ \dot{u} \end{pmatrix} = \begin{pmatrix} 0 & A^{1/2} \\ -A^{1/2} & -B \end{pmatrix}\begin{pmatrix} A^{1/2}u \\ \dot{u} \end{pmatrix}. \tag{1.3}$$

Let $V = \mathscr{D}(A^{\frac{1}{2}})$, $\mathscr{H} = V \times H$ with the naturally induced inner products. Then, equation (1.2) is equivalent to the first order equation in $\mathscr{H}$

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} u \\ \dot{u} \end{pmatrix} = \mathscr{A}_B \begin{pmatrix} u \\ \dot{u} \end{pmatrix}, \tag{1.4}$$

where

$$\begin{cases} \mathscr{A}_B = \begin{pmatrix} 0 & I \\ -A & -B \end{pmatrix}, \\ \mathscr{D}(\mathscr{A}_B) = \mathscr{D}(A) \times [\mathscr{D}(A^{1/2}) \cap \mathscr{D}(B)]. \end{cases} \tag{1.5}$$

Chen and Russell [1] conjectured that $\mathscr{A}_B$ is the infinitesimal generator of an analytic semigroup on $\mathscr{H}$ if

$$\mathscr{D}(A^{1/2}) \subset \mathscr{D}(B), \tag{1.6}$$

and either of the following two inequalities holds for some constants $\beta_1, \beta_2 > 0$:

$$\beta_1(A^{1/2}v,\, v) \le (Bv,\, v) \le \beta_2(A^{1/2}v,\, v), \quad v \in \mathscr{D}(A^{1/2}); \tag{1.7}$$

$$\beta_1(Av,\, v) \le (B^2v,\, v) \le \beta_2(Av,\, v), \quad v \in \mathscr{D}(A). \tag{1.8}$$

In [2,3], Huang gave the complete proofs of the two conjectures. Then, the spectral property of this systems and some fundamental results for the holomorphic property and exponential stability of the semigroup associated with these systems were discussed in [4-12].

In [13], a linear second order evolution equation in the frame of Banach spaces was explored, which can model the elastic systems with structural damping, new forms of the corresponding first order evolution equation were introduced, and sufficient conditions for analyticity and exponential stability of the associated semigroups were given. In [14], the existence results of mild solution for the elastic systems with structural damping were established by the same authors as in [13], firstly via the contraction mapping principle, secondly via the Schauder fixed theorem. In addition, by applying the method of upper and lower solutions and the operator semigroups theory, the existence results of extremal mild solutions and the asymptotic stability of solutions for the initial-value problem (1.1) were obtained, see [15,16].

In this paper, we will investigate the global existence of mild solutions for the initial-value problem (1.1) in Banach space $\mathbb{X}$, via a fixed point analysis approach. By means of the Leray-Schauder fixed point theorem, we derive conditions under which a solution exists globally. The result will improve and extend some relevant results in elastic systems with structural damping.