Ann. of Appl. Math.
35:2(2019), 197-211

EXISTENCE OF PERIODIC SOLUTION FOR A KIND OF (m, n)-ORDER GENERALIZED NEUTRAL DIFFERENTIAL EQUATION*†

Shaowen Yao‡

(*School of Math. and Information Science, Henan Polytechnic University, Jiaozuo 454000, Henan, PR China*)

Abstract

In this paper, we consider the following high-order p-Laplacian generalized neutral differential equation with variable parameter

$$(\varphi_p(x(t)-c(t)x(t-\sigma))^{(n)})^{(m)}+g(t,x(t),x(t-\tau(t)),x'(t),\cdots,x^{(m)}(t))=e(t).$$

By the coincidence degree theory and some analysis skills, sufficient conditions for the existence of periodic solutions are established.

Keywords periodic solution; p-Laplacian; high-order; neutral operator; variable parameter

2000 Mathematics Subject Classification 34C25; 34K13; 34K40

1 Introduction

In this paper, we consider the following high-order p-Laplacian neutral differential equation with variable parameter

$$(\varphi_p(x(t)-c(t)x(t-\sigma))^{(n)})^{(m)}+g(t,x(t),x(t-\tau(t)),x'(t),\cdots,x^{(n)}(t))=e(t), \quad (1.1)$$

where $p>1$, $\varphi_p(x)=|x|^{p-2}x$ for $x\neq 0$ and $\varphi_p(0)=0$; $g:\mathbb{R}^{n+2}\to\mathbb{R}$ is a continuous periodic function with $g(t+T,\cdot,\cdots,\cdot)\equiv g(t,\cdot,\cdots,\cdot)$, and $g(t,a,a,0,\cdots,0)-e(t)\not\equiv 0$ for all $a\in\mathbb{R}$. $e:\mathbb{R}\to\mathbb{R}$ is a continuous periodic function with $e(t+T)\equiv e(t)$ and $\int_0^T e(t)\mathrm{d}t=0$; $c\in C^n(\mathbb{R},\mathbb{R})$ and $c(t+T)\equiv c(t)$, T is a positive constant; n and m are positive integers.

Complicated behavior of technical applications is often modeled by nonlinear high-order differential equations [1], for example, the Lorenz model of a simplified hydrodynamic flow, the dynamo model of erratic inversion of the earth's magnetic

*Research is supported by National Natural Science Foundation of China (71601072).

†Manuscript received July 20, 2018

‡Corresponding author. E-mail: yaoshaowen_78@126.com

field, etc. Oftentimes high-order equations are a result of combinations of lower order equations. Due to its obvious complexity, studies on high-order differential equation are rather few, especially on high-order neutral differential equation. In recent years, there has been many perfect results on periodic solutions for high-order differential equations (see [2-11] and the references cited therein). For example, in [8], Pan studied the nth-order differential equation

$$x^{(n)}(t)=\sum_{i=1}^{n-1}b_ix^{(i)}(t)+f(t,x(t),x(t-\tau_1(t)),\cdots,x(t-\tau_m(t)))+p(t), \tag{1.2}$$

and obtained the existence of periodic solutions for (1.2). In [7], Li and Lu considered the following high-order p-Laplacian differential equation

$$(\varphi_p(y^{(m)}(t)))^{(m)}=f(y(t))y'(t)+h(y(t))+\beta(t)g(y(t-\tau(t)))+e(t), \tag{1.3}$$

and using the theory of Fourier series, Bernoulli number theory and continuation theorem of coincidence degree theory, they studied the existence of periodic solutions for (1.3). Afterwards, Wang and Lu [9] investigated the existence of periodic solution for the high-order neutral functional differential equation with distributed delay

$$(x(t)-cx(t-\sigma))^{(n)}+f(x(t))x'(t)+g\left(\int_{-r}^{0}x(t+s)\mathrm{d}\alpha(s)\right)=p(t). \tag{1.4}$$

Recently, in [10] and [11], Ren, Cheng and Cheung observed a high-order p-Laplacian neutral differential equation

$$(\varphi_p(x(t)-cx(t-\sigma))^{(l)})^{(n-l)}=F(t,x(t),x'(t),\cdots,x^{(l-1)}(t)), \tag{1.5}$$

and presented sufficient conditions for the existence of periodic solutions for (1.5) in the critical case (that is, $|c|=1$) and in the general case (that is, $|c|\neq 1$), respectively.

Inspired by these results, we consider a generalized high-order neutral differential equation with variable parameter (1.1). Here $A=x(t)-c(t)x(t-\sigma)$ is a natural generalization of the operator $A_1=x(t)-cx(t-\sigma)$, which typically possesses a more complicated nonlinearity than A_1. For example, A_1 is homogeneous in the following sense $(A_1x)'(t)=(A_1x')(t)$, whereas A in general is inhomogeneous. As a consequence many new results of differential equations with the neutral operator A will not be direct generalizations of known theorems of neutral differential equations.

The paper is organized as follows: In Section 2, we first give qualitative properties of the neutral operator A which can be helpful to study differential equations with operator; in Section 3, based on Mawhin's continuation theory and some new inequalities, we obtain sufficient conditions for the existence of periodic solutions for (1.1), also an example is also given to illustrate our results.