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HOPF BIFURCATION ANALYSIS IN A MONOD-HALDANE PREDATOR-PREY MODEL WITH THREE DELAYS*

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Abstract

We analysis Hopf bifurcation in a Monod-Haldane predator-prey model with three delays in this paper. Fixing τ_1 and τ_2 and taking τ_3 as parameter, the direction and stability of Hopf bifurcation are studied by using center manifold theorem and normal form. At last some simulations are given to support our results.

Keywords Hopf bifurcation; Monod-Haldane predator-prey model; delays
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1 Introduction

The dynamics of predator-prey model with delay have attract many interest for researchers [1-8], For example [3, 8] discussed the effect of delay on the global stability for predator-prey system, [6] discussed Hopf bifurcation of a ratio-dependent predator-prey system with two delays, besides [4] studied Hopf bifurcation of delayed predator-prey model with stage-structure for prey.

We know that Holling functional response is usually used to represent the grasping for predator and the functional responses is usually monotonic. But in microbial dynamics or chemical kinetics, the functional response represents the uptake of substance by the microorganisms, and the nonmonotonic responses occur by experiment. For example the inhibitory effect on the growth rate occurs when the nutrient concentration reaches a sufficient level [9]. This case always exists when micro-organisms are used for waste decomposition or for water purification [11]. The response function $p(x) = \frac{mx}{a+bx+x^2}$ is proposed by Ardrew to present the inhibitory effect which is called Monod-Haldane function [10] at hight concentration. Besides Sokol and

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Howell [12] proposed a simple Monod-Haldane function of form $p(x) = \frac{mx}{a+x^2}$ to describe the uptake of phenol by a pure culture of Pseudomonas putida growing on phenol in a continuous culture. In [13] the ability of predator for prey is expressed by simplified Monod-Haldane function $p(x) = \frac{x}{a+x^2}$. So we think Monod-Haldane function in predator-prey model should be more realistic in some situation. Besides the diffusion between patch is introduced into predator-prey model should be more reasonable. As we know, some kinds of delays always exist during the works about predator-prey system, such as the hunting delay for predator, the delay caused by gestation or maturation for predator and so on. Recently the following predator-prey system [21] with three delays was studied:

$$\begin{cases} \dot{x}_1(t) = x_1(t)\Big(r_1 - a_1x_1(t-\tau_1) - \dfrac{c_1y(t-\tau_3)}{1+kx_1^2(t)}\Big) + \delta(x_2(t) - x_1(t)), \\ \dot{x}_2(t) = x_2(t)(r_2 - a_2x_2(t)) + \delta(x_1(t) - x_2(t)), \\ \dot{y}(t) = y(t)\Big(d_1 + \dfrac{c_2x_1(t)}{1+kx_1^2(t)} - d_2y(t-\tau_2)\Big), \end{cases} \tag{1.1}$$

with the initial condition:

$$x_i(\theta) > 0, \quad y(\theta) > 0, \quad \theta \in [-\tau, 0], \quad i = 1, 2, \quad \tau = \max\{\tau_1, \tau_2, \tau_3\}, \tag{1.2}$$

where $x_1(t), x_2(t)$ denote the numbers of prey species in patch 1 and patch 2 respectively, $y(t)$ denotes the numbers of predator species in patch 1, c_1 is the capture rate. Monod-Haldane response function $\frac{x_1}{1+kx_1^2}$ expresses the capture ability of predator, c_2 denotes the conversion rate, $r_i\,(i = 1, 2)$ is the birth rate of prey species in patch i respectively, $a_i\,(i = 1, 2)$ and d_2 are the coefficients of intra-specific competition, d_1 is the birth rate for predator, the delays τ_1, τ_2 represent negative feedback of prey and predator in patch 1 respectively, τ_3 is the hunting delay. δ is the diffusion coefficient.

Assuming $\tau_3 = 0$, the author [2] studied the Hopf bifurcation of system (1.1) with two delays (τ_1 and τ_2) under four cases: (1) $\tau_1 \neq 0$, $\tau_2 = 0$, (2) $\tau_1 = 0$, $\tau_2 \neq 0$, (3) $\tau_1 = \tau_2 = \tau \neq 0$, (4) $\tau_1 \neq \tau_2$, $\tau_1 \in (0, \tau_{10})$, $\tau_2 > 0$. But delay τ_3 always exists in reality, we should consider its importance for dynamics. Although there are many works about Hopf bifurcation with two delays [6,14-17]. But in my opinion, the Hopf bifurcations with three delays [19,20] are rarely. In [20], the author considered the following model HIV-1 system with three delays:

$$\begin{cases} \dot{x} = \lambda - dx(t) - \beta x(t)v(t), \\ \dot{y} = \beta \mathrm{e}^{-a\tau_1}x(t-\tau_1)v(t-\tau_1) - ay(t) - \alpha w(t)y(t), \\ \dot{z} = \alpha w(t)y(t) - bz(t), \\ \dot{v} = k\mathrm{e}^{-a_2\tau_2}y(t-\tau_2) - pv(t), \\ \dot{w} = c\mathrm{e}^{-a_3\tau_3}z(t-\tau_3) - qw(t). \end{cases} \tag{1.3}$$