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EXTINCTION OF A DISCRETE COMPETITIVE SYSTEM WITH BEDDINGTON-DEANGELIS FUNCTIONAL RESPONSE AND THE EFFECT OF TOXIC SUBSTANCES*†

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Abstract

In this paper, we consider a discrete competitive system with Beddington-DeAngelis functional response and the effect of toxic substances. By constructing some suitable Lyapunov type extinction functions, sufficient conditions which guarantee the extinction of a species and global attractivity of the other one are obtained. Our results not only supplement and improve but also generalize some existing ones. Numerical simulations show the feasibility of our results.

Keywords discrete competitive system; Beddington-DeAngelis functional response; extinction; toxic substances

2000 Mathematics Subject Classification 34D20

1 Introduction

In the paper, we are concerned with the extinction property of the following discrete competitive phytoplankton system with Beddington-DeAngelis functional response and the effect of toxic substances:

$$\begin{cases} x_1(n+1)=x_1(n)\exp\Big\{r_1(n)-a_1(n)x_1(n)-\dfrac{b_1(n)x_2(n)}{\alpha_1(n)+\beta_1(n)x_1(n)+\gamma_1(n)x_2(n)} \\ \qquad\qquad -c_1(n)x_1(n)x_2(n)\Big\}, \\ x_2(n+1)=x_2(n)\exp\Big\{r_2(n)-a_2(n)x_2(n)-\dfrac{b_2(n)x_1(n)}{\alpha_2(n)+\beta_2(n)x_1(n)+\gamma_2(n)x_2(n)} \\ \qquad\qquad -c_2(n)x_1(n)x_2(n)\Big\}, \end{cases} \tag{1.1}$$

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where $x_1(n)$, $x_2(n)$ are population densities of species x_1 and x_2 at the nth generation, respectively. $r_i(n)$ $(i=1,2)$ are the intrinsic growth rates of species; $a_i(n)$ $(i=1,2)$ are the rates of intraspecific competition of the first and second species; the interspecific competition between two species takes the Beddington-DeAngelis functional response type

$$\frac{b_1(n)x_2(n)}{\alpha_1(n)+\beta_1(n)x_1(n)+\gamma_1(n)x_2(n)} \quad \text{and} \quad \frac{b_2(n)x_1(n)}{\alpha_2(n)+\beta_2(n)x_1(n)+\gamma_2(n)x_2(n)},$$

respectively. The terms $c_i(n)x_1(n)x_2(n)$ $(i=1,2)$ denote the effect of toxic substances, under the assumption that each species produces a substance toxic to the other, only when the other is present.

We will also consider the extinction property of the following two species nonautonomous competitive phytoplankton system with Beddington-DeAngelis functional response:

$$\begin{cases} x_1(n+1)=x_1(n)\exp\Big\{r_1(n)-a_1(n)x_1(n)-\dfrac{b_1(n)x_2(n)}{\alpha_1(n)+\beta_1(n)x_1(n)+\gamma_1(n)x_2(n)} \\ \qquad\qquad -c_1(n)x_1(n)x_2(n)\Big\}, \\ x_2(n+1)=x_2(n)\exp\Big\{r_2(n)-a_2(n)x_2(n)-\dfrac{b_2(n)x_1(n)}{\alpha_2(n)+\beta_2(n)x_1(n)+\gamma_2(n)x_2(n)}\Big\}, \end{cases} \tag{1.2}$$

where all the coefficients have the same meaning as those of system (1.1). However, we assume that the second species could produce toxic, while the first one is nontoxic producing.

Motivated by Gopalsamy [1], Qin, Liu and Chen [2] studied the following discrete competitive system:

$$\begin{cases} x_1(n+1)=x_1(n)\exp\left\{r_1(n)-a_1(n)x_1(n)-\dfrac{b_1(n)x_2(n)}{1+x_2(n)}\right\}, \\ x_2(n+1)=x_2(n)\exp\left\{r_2(n)-a_2(n)x_2(n)-\dfrac{b_2(n)x_1(n)}{1+x_1(n)}\right\}, \end{cases} \tag{1.3}$$

which is a special case of system (1.1) with $\alpha_i(n)=\gamma_1(n)=\beta_2(n)=1$ and $\beta_1(n)=\gamma_2(n)=c_i(n)=0$ $(i=1,2)$. Qin et al. [2] obtained sufficient conditions on the permanence and global asymptotic stability of positive periodic solutions of system (1.3). Wang and Liu [3] further considered the existence, uniqueness and uniformly asymptotic stability of positive almost periodic solution of system (1.3) with almost periodic parameters. Extinction of a species and the stability property of another one were considered in [4]. Noting that ecosystems in the real world are often distributed by unpredictable forces that can result in changes in biological parameters, Wang et al. [5] considered system (1.3) with feedback controls and established a criterion for the existence and uniformly asymptotic stability of unique positive almost periodic