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EXACT SOLUTIONS OF (2+1)-DIMENSIONAL NONLINEAR SCHRÖDINGER EQUATION BASED ON MODIFIED EXTENDED TANH METHOD *

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Abstract

In this paper, the modified extended tanh method is used to construct more general exact solutions of a (2+1)-dimensional nonlinear Schrödinger equation. With the aid of Maple and Matlab software, we obtain exact explicit kink wave solutions, peakon wave solutions, periodic wave solutions and their 3D images.

Keywords nonlinear Schrödinger equation; modified extended tanh method; kink wave solution; peakon wave solution; periodic wave solution

2000 Mathematics Subject Classification 81Q05; 35Q41

1 Introduction

It is well known that Schrödinger equation is one of the most basic equation of quantum mechanics. It reflects the state of micro particle changing with time. As it is a powerful tool for solving non relativistic problems in atomic physics, it has been widely used in the field of atomic, molecular, solid state physics, nuclear physics, chemistry and so on. Recently, searching and constructing exact solutions of nonlinear partial differential (NLPD) equation is very meaningful for it can describe the problems of mechanics, control process, ecological and economic system, chemical recycling system and epidemiological.

In the past several decades, much efforts have been made on this aspect and many useful methods have been proposed such as inverse scattering method, Jacobi elliptic function method, F-expansion method, Darboux transform, the sine-cosine method and the tanh method and so on [1-18]. Among them, the tanh method is

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widely used as it can find exact as well as approximate solutions in a systematic way. Subsequently, Fan has proposed an extended tanh method and obtained the travelling wave solutions that can not be obtained by the tanh method. Based on this approach, we employ the modified extended tanh method to construct a series of exact travelling wave solutions of a (2+1)-dimensional nonlinear Schrödinger (NLS) equation as

$$iu_t + \alpha u_{xx} + \beta u_{yy} + r|u^2|u = 0. \tag{1}$$

The rest of this paper is organized as follows. In Section 2, we shall introduce the modified extended tanh method. In Section 3, we illustrate this method in detail with the (2+1)-dimensional nonlinear Schrödinger (NLS) equation. In Section 4, the image simulations of exact Travelling Wave Solutions of (1) are given. Finally, a short conclusion is given in Section 4.

2 The Modified Extended Tanh Method

In this section, we review the modified extended tanh method.

The modified extended tanh method is developed by Malflied in [10,11], and used in [12-14] among many others. Since all derivatives of a tanh can represented by tanh itself, we consider the general NLPDE in two variables

$$H(u, u_t, u_x, u_x x, u_t t, u_x t, \cdots) = 0.$$

Now we consider its travelling $u(x,t) = u(\xi)$, where $\xi = x - ct$ or $\xi = x + ct$ and the equation becomes an ordinary differential equation. We apply the following series expansion

$$u(\xi) = \sum_{i=0}^{N} a_i \phi^i + \sum_{i=1}^{N} b_i \phi^{-i}, \quad \phi' = b + \phi^2,$$

where b is a parameter to be determined, $\phi = \phi(\xi)$ and $\phi' = \frac{\mathrm{d}\phi}{\mathrm{d}\xi}$.

To determine the parameter N, we usually balance the linear terms of highest-order in the resulting equation with the highest-order nonlinear terms. Then we can get all coefficients of different powers of ϕ and determine a_i, b_i, b, c by making them equal to zeros.

The Riccati equation has the following general solutions:

(a) If $b < 0$, $\phi = -\sqrt{-b}\tanh(\sqrt{-b}\xi)$;

(b) if $b > 0$, $\phi = \sqrt{b}\tan(\sqrt{b}\xi)$;

(c) if $b = 0$, $\phi = -1/\xi$.

3 Exact Travelling Wave Solutions of (1)

We consider the travelling wave solution $u(x,y,t) = u(\xi)$, $\xi = x + ky - ct$ of (1), and also

$$u(\xi) = P(\xi) + iQ(\xi). \tag{2}$$

According to (1) and (2), we can get

$$i(P_t + iQ_t) + \alpha(P_{xx} + iQ_{xx}) + \beta(P_{yy} + iQ_{yy}) + r(P^2 + Q^2)(P + iQ) = 0. \tag{3}$$

From (3) we can obtain the following two equation

$$-Q_t + \alpha P_{xx} + \beta P_{yy} + rP^3 + rQ^2P = 0, \tag{4a}$$

$$P_t + \alpha Q_{xx} + \beta Q_{yy} + rP^2Q + rQ^3 = 0. \tag{4b}$$

For $u(x, y, t) = u(\xi)$ and $\xi = x + ky - ct$, we can transform (4) into ordinary differential equations

$$cQ' + (\alpha + \beta k^2)P'' + rP^3 + rQ^2P = 0, \tag{5a}$$

$$-cP' + (\alpha + \beta k^2)Q'' + rP^2Q + rQ^3 = 0. \tag{5b}$$

The solution can be expressed as the following form

$$P(\xi) = \sum_{i=0}^{N} a_i\phi^i + \sum_{i=1}^{N} b_i\phi^{-i}. \tag{6a}$$

$$Q(\xi) = \sum_{i=0}^{N1} c_i\phi^i + \sum_{i=1}^{N1} d_i\phi^{-i}. \tag{6b}$$

Balancing the linear term of highest order with the nonlinear term in both equations, we find

$$N - 2 + 4 = 2N + N_1 = 3N,$$
$$N_1 - 2 + 4 = 2N_1 + N = 3N.$$

Thus, $N = N_1 = 1$, and

$$P(\xi) = a_0 + a_1\phi + b_1\phi^{-1}, \tag{7a}$$

$$Q(\xi) = c_0 + c_1\phi + d_1\phi^{-1}. \tag{7b}$$

With $\phi' = b + \phi^2$, we get

$$P'(\xi) = (a_1b - b_1) + a_1\phi^2 - b_1b\phi^{-2}, \tag{8a}$$

$$P''(\xi) = 2a_1\phi^3 + 2ba_1\phi + 2b^2b_1\phi^{-3} + 2bb_1\phi^{-1}, \tag{8b}$$

$$Q'(\xi) = (c_1b - d_1) + c_1\phi^2 - d_1b\phi^{-2}, \tag{8c}$$

$$Q''(\xi) = 2c_1\phi^3 + 2bc_1\phi + 2b^2d_1\phi^{-3} + 2bd_1\phi^{-1}. \tag{8d}$$

Substitute (7) and (8) into the ordinary differential equations (5a) and (5b), then we obtain the coefficients of ϕ^0, ϕ, ϕ^2, ϕ^3, ϕ^{-1}, ϕ^{-2} and ϕ^{-3}, respectively,

ϕ^0: $-c(a_1b - b_1) + r(a_0^2 + 2a_1b_1)c_0 + 2ra_0b_1c_1 + 2ra_0a_1d_1 + rc_0(c_0^2 + 2c_1d_1) + 4rc_0c_1d_1 = 0$,

ϕ: $2(\alpha + \beta k^2)c_1b + rc_1(a0^2 + 2a_1b_1) + 2rc_0a_0a_1 + ra_1^2d_1 + 2rc_0^2c_1 + rc_1(c0^2 + 2c_1d_1) + rd_1c_1^2 = 0$,

ϕ^2: $-ca_1 + rc_0a_1^2 + 2ra_0a_1c_1 + 3rc_0c_1^2 = 0$,

ϕ^3: $2c_1(\alpha + \beta k^2) + rc_1a_1^2 + rc_1^3 = 0$,

ϕ^{-1}: $2d_1b(\alpha+\beta k^2)+2ra_0c_0b_1+rc_1b_1^2+rd_1(a_0^2+2a_1b_1)+2rc_0^2d_1+rc_1d_1^2+rd_1(c_0^2+2c_1d_1)=0$,

ϕ^{-2}: $cb_1b+rc_0b_1^2+2ra_0b_1d_1+3rc_0d_1^2=0$,

ϕ^{-3}: $2d_1b^2(\alpha+\beta k^2)+rd_1b_1^2+rd_1^3=0$

and

ϕ^0: $c(c_1b-d_1)+ra_0(a_0^2+2a_1b_1)+4ra_0a_1b_1+ra_0(c_0^2+2c_1d_1)+2ra_1c_0d_1+2rc_0c_1b_1=0$,

ϕ: $2a_1b(\alpha+\beta k^2)+2ra_0^2a_1+ra_1(a_0^2+2a_1b_1)+ra_1^2b_1+2ra_0c_0c_1+ra_1(c_0^2+2c_1d_1)+rb_1c_1^2=0$,

ϕ^2: $cc_1+ra_1^2a_0+2ra_0a_1^2+2ra_1c_0c_1+ra_0c_1^2=0$,

ϕ^3: $2a_1(\alpha+\beta k^2)+ra_1^3+ra_1c_1^2=0$,

ϕ^{-1}: $2b_1b(\alpha+\beta k^2)+2ra_0^2b_1+ra_1b_1^2+rb_1(a_0^2+2a_1b_1)+ra_1d_1^2+2ra_0c_0d_1+rb_1(c_0^2+2c_1d_1)=0$,

ϕ^{-2}: $-cbd_1+ra_0b_1^2+2ra_0b_1^2+ra_0d_1^2+2rc_0b_1d_1=0$,

ϕ^{-3}: $2b_1b^2(\alpha+\beta k^2)+rb_1^3+rb_1d_1^2=0$.

With the aid of Maple, we obtain $a_0,a_1,b_1,c_0,c_1,d_1,b,c,k$ as follows.

Case (1)

$$a_0=-c_0c_1\Big(-\frac{2\alpha+2\beta k^2+rc_1^2}{r}\Big)^{-\frac{1}{2}},\quad a_1=\Big(-\frac{2\alpha+2\beta k^2+rc_1^2}{r}\Big)^{\frac{1}{2}},$$
$$b_1=-\frac{c_0^2}{4}\Big(-\frac{2\alpha+2\beta k^2+rc_1^2}{r}\Big)^{-\frac{1}{2}},\quad b=-\frac{1}{4}\frac{c_0^2r}{2\alpha+2\beta k^2+rc_1^2},$$
$$c=-2c_0(\alpha+\beta k^2)\Big(-\frac{2\alpha+2\beta k^2+rc_1^2}{r}\Big)^{-\frac{1}{2}},\quad c_0=c_0,$$
$$c_1=c_1,\quad d_1=\frac{1}{4}\frac{c_0^2c_1r}{2\alpha+2\beta k^2+rc_1^2},\quad k=k,$$

where r,β,α,k,c_0,c_1 are arbitrary constants.

Case (2)

$$a_0=-d_1\Big(-\frac{2\alpha+2\beta k^2}{rd_1^2+rb_1^2}\Big)^{\frac{1}{4}},\quad a_1=0,\quad b_1=b_1,\quad b=\Big(-\frac{2\alpha+2\beta k^2}{rd_1^2+rb_1^2}\Big)^{-\frac{1}{4}},$$
$$c=-r(d_1^2+b_1^2)\Big(-\frac{2\alpha+2\beta k^2}{rd_1^2+rb_1^2}\Big)^{\frac{3}{4}},\quad c_0=b_1\Big(-\frac{2\alpha+2\beta k^2}{rd_1^2+rb_1^2}\Big)^{\frac{1}{4}},$$
$$c_1=0,\quad d_1=d_1,\quad k=k,$$

where r,β,α,k,b_1,d_1 are arbitrary constants.

Case (3)

$$a_0=\Big(-\frac{2\alpha d_1^2+2d_1^2\beta k^2}{r}\Big)^{\frac{1}{4}},\quad a_1=0,\quad b_1=0,\quad b=d_1^2\Big(-\frac{2\alpha d_1^2+2d_1^2\beta k^2}{r}\Big)^{-\frac{1}{2}},$$

$$c=\frac{r}{d_1}\Big(-\frac{2\alpha d_1^2+2d_1^2\beta k^2}{r}\Big)^{\frac{3}{4}},\quad c_0=0,\quad c_1=0,\quad d_1=d_1,\quad k=k,$$

where r,β,α,k,d_1 are arbitrary constants.

Case (4)

$$a_0=a_0,\quad a_1=0,\quad b_1=0,\quad b=-\frac{1}{2}\frac{ra_0^2}{\alpha+\beta k^2},\quad c=-ra_0\Big(-\frac{2\alpha+2\beta k^2}{r}\Big)^{\frac{1}{2}},$$
$$c_0=0,\quad c_1=\Big(-\frac{2\alpha+2\beta k^2}{r}\Big)^{\frac{1}{2}},\quad d_1=0,\quad k=k,$$

where r,β,α,k,a_0 are arbitrary constants.

Case (5)

$$a_0=a_0,\quad a_1=0,\quad b_1=0,\quad b=-\frac{1}{8}\frac{ra_0^2}{\alpha+\beta k^2},\quad c=-ra_0\Big(-\frac{2\alpha+2\beta k^2}{r}\Big)^{\frac{1}{2}},$$
$$c=-ra_0\Big(-\frac{4+2\beta k^2}{r}\Big)^{\frac{1}{2}},\quad c_0=0,\quad c_1=\Big(-\frac{2\alpha+2\beta k^2}{r}\Big)^{\frac{1}{2}},$$
$$d_1=-\frac{a_0^2}{4}\Big(-\frac{2\alpha+2\beta k^2}{r}\Big)^{-\frac{1}{2}},\quad k=k,$$

where r,β,α,k,a_0 are arbitrary constants.

Case (6)

$$a_0=0,\quad a_1=\Big(-\frac{2\alpha+2\beta k^2}{r}\Big)^{\frac{1}{2}},\quad b_1=0,\quad b=-\frac{1}{2}\frac{rc_0^2}{\alpha+\beta k^2},$$
$$c=rc_0\Big(-\frac{2\alpha+2\beta k^2}{r}\Big)^{\frac{1}{2}},\quad c_0=c_0,\quad c_1=0,\quad d_1=0,\quad k=k,$$

where r,β,α,k,c_0 are arbitrary constants.

Case (7)

$$a_0=-c_0c_1\Big(-\frac{2\alpha+2\beta k^2+rc_1^2}{2}\Big)^{-\frac{1}{2}},\quad a_1=\Big(-\frac{2\alpha+2\beta k^2+rc_1^2}{2}\Big)^{\frac{1}{2}},\quad b_1=0,$$
$$b=\frac{-c_0^2r}{2\alpha+2\beta k^2+rc_1^2},\quad c=-2(\alpha+\beta k^2)c_0\Big(-\frac{2\alpha+2\beta k^2+rc_1^2}{2}\Big)^{-\frac{1}{2}},$$
$$c_0=c_0,\quad c_1=c_1,\quad d_1=0,\quad k=k,$$

where r,β,α,k,c_0,c_1 are arbitrary constants.

Case (8)

$$a_0=0,\quad a_1=\Big(-\frac{2\alpha+2\beta k^2}{r}\Big)^{\frac{1}{2}},\quad b_1=\frac{1}{8}\frac{rc_0^2}{\alpha+\beta k^2}\Big(-\frac{2\alpha+2\beta k^2}{r}\Big)^{\frac{1}{2}},$$
$$b=-\frac{1}{8}\frac{rc_0^2}{\alpha+\beta k^2},\quad c=rc_0\Big(-\frac{2\alpha+2\beta k^2}{r}\Big)^{\frac{1}{2}},\quad c_0=c_0,\quad c_1=0,\quad d_1=0,\quad k=0,$$

where r,β,α,k,c_0 are arbitrary constants.

Case (9)

$$a_0 = 0, \quad a_1 = 0, \quad b_1 = c_0^2\sqrt{-\frac{r}{2\alpha + 2\beta k^2}}, \quad b = -\frac{1}{2}\frac{c_0^2 r}{\alpha + \beta k^2},$$
$$c = -\frac{c_0 r}{\sqrt{-\frac{r}{2\alpha+2\beta k^2}}}, \quad k = k, \quad c_0 = c_0, \quad c_1 = 0, \quad d_1 = 0,$$

where r, β, α, k, c_0 are arbitrary constants.

If $b > 0$, we get

$$u = a_0 + a_1 v(x,y,t) + b_1 v(x,y,t)^{-1} + i(c_0 + c_1 v(x,y,t) + d_1 v(x,y,t)^{-1}), \tag{9a}$$

where $v(x,y,t) = \sqrt{b}\tan(\sqrt{b}(x + ky - ct))$.

If $b = 0$, we get

$$u = a_0 + a_1 v(x,y,t) + b_1 v(x,y,t)^{-1} + i(c_0 + c_1 v(x,y,t) + d_1 v(x,y,t)^{-1}), \tag{9b}$$

where $v(x,y,t) = \frac{1}{x+ky-ct}$.

If $b < 0$, we get

$$u = a_0 + a_1 v(x,y,t) + b_1 v(x,y,t)^{-1} + i(c_0 + c_1 v(x,y,t) + d_1 v(x,y,t)^{-1}), \tag{9c}$$

where $v(x,y,t) = -\sqrt{-b}\tanh(\sqrt{-b}(x + ky - ct))$.

Substituting all those situation into (9) respectively, we can get all solutions of the derivative nonlinear Schrödinger equation.

4 Image Simulation

In order to grasp these exact travelling solutions, we choose several exact solutions and use the Matlab software to simulate images. In the process of image simulation, the figures and values of parameters we selected are shown as follows.

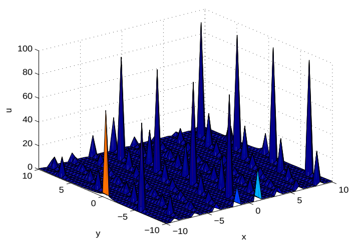

Figure 1: 3-dimensional wave of Case (1).

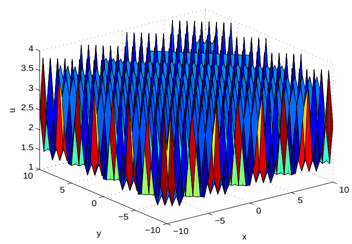

Figure 2: 3-dimensional wave of Case (2).

In Figure 1, we take $r = -1$, $\beta = 11$, $k = 0.1$, $c_0 = -0.25$, $c_1 = 0.1$, $\alpha = -0.2$, $t = 0.1$. In Figure 2, we take $r = -6$, $\beta = -3$, $k = 1$, $d_1 = 1$, $\alpha = 1$, $b_1 = -1$, $t = 0.1$.

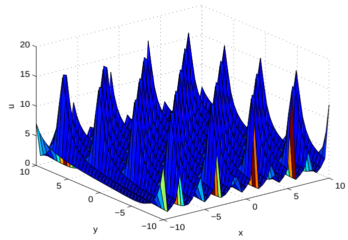

Figure 3: 3-dimensional wave of Case (3).

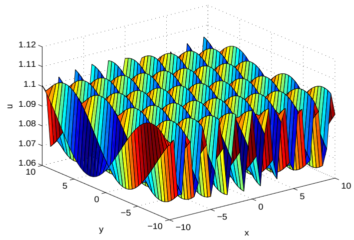

Figure 4: 3-dimensional wave of Case (3).

In Figure 3, we take $r = 0.1$, $\beta = 0.3$, $k = 0.1$, $d_1 = -2$, $\alpha = 0.2$, $t = 0.1$. In Figure 4, we take $r = -6$, $\beta = 2$, $k = 0.1$, $c_0 = 6$, $\alpha = -4$, $d_1 = 2$, $t = 0.1$.

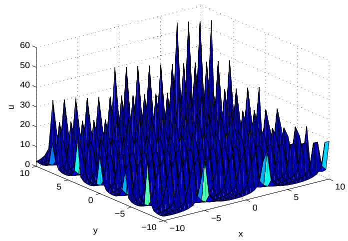

Figure 5: 3-dimensional wave of Case (4).

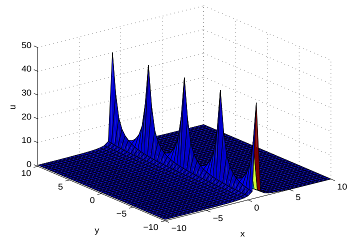

Figure 6: 3-dimensional wave of Case (5).

In Figure 5, we take $r = -1$, $\beta = 3$, $k = 3$, $a_0 = 2$, $\alpha = -0.7$, $t = 0.1$. In Figure 6, we take $r = -1$, $\beta = -0.3$, $k = 0.1$, $a_0 = -0.25$, $\alpha = -1$, $t = 0.1$.

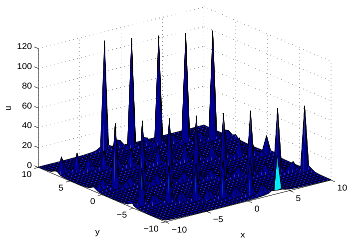

Figure 7: 3-dimensional wave of Case (6).

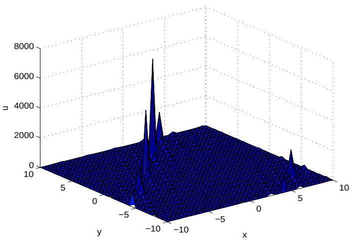

Figure 8: 3-dimensional wave of Case (7).

In Figure 7, we take $r = -6$, $\beta = 3$, $k = 2.5$, $c_0 = -0.5$, $\alpha = -2$, $t = 0.1$. In Figure 8, we take $r = -1$, $\beta = 3$, $k = 2.9$, $c_0 = 6$, $\alpha = -2$, $c_1 = 2$, $t = 0.1$.

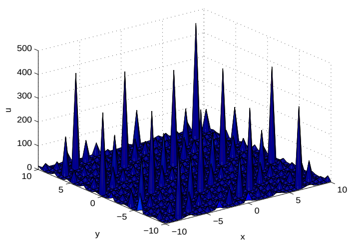

Figure 9: 3-dimensional wave of Case (8).

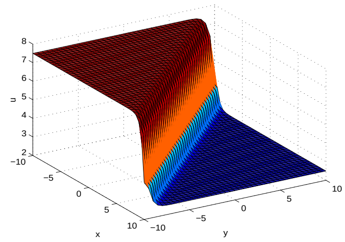

Figure 10: 3-dimensional wave of Case (9).

In Figure 9, we take $r = -2$, $\beta = 2$, $k = 1.6$, $c_0 = 2.5$, $\alpha = 3.2$, $t = 0.1$. In Figure 10, we take $r = -0.1$, $\beta = 1$, $k = 1$, $c_0 = 2.5$, $\alpha = -1$, $c_1 = 1$, $t = 0.1$.

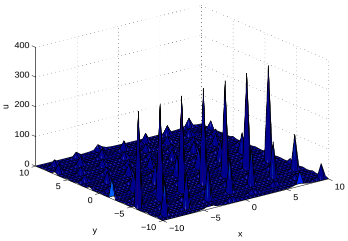

Figure 11: 3-dimensional wave of Case (9).

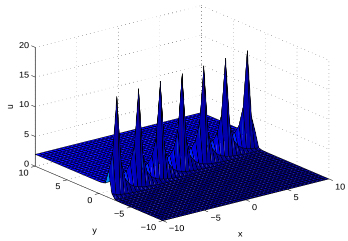

Figure 12: 3-dimensional wave of Case (9).

In Figure 11, we take $r = -6$, $\beta = 1$, $k = -6$, $c_0 = 1$, $\alpha = -1$, $t = 0.1$. In Figure 12, we take $r = 6$, $\beta = 1$, $k = -6$, $c_0 = 1$, $\alpha = -1$, $t = 0.01$.

5 Conclusion

In this paper we study the nonlinear Schrödinger equation by finding its exact travelling wave solutions through the modified extended tanh method. With the aid of waveform graphs of the solutions, we can obtain the related properties of the equation. In addition, we can also use other methods to obtain the bounded solutions. However, the modified extended tanh method is more concise, more direct and simpler than any other existing methods.

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