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Flexible Multibody System Dynamics by Means of a Spectral Based Meshless Approach

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Abstract. It is a common practice in industry to model the elasticity in flexible multibody dynamics, when the deformations are small, by means of a linear finite element approach and of a model condensation strategy. Taking into account the flexibility in multibody modelling may require computationally expensive numerical models to be managed. Proper shape functions are introduced in this paper to model the displacements of flexible slender beam components, without the need of any spatial discretization; a novel formulation of the flexible properties of beam-like components follows and a small size motion equation set can be obtained. Modelling aspects, from point location to constraint equations and to elastodynamic modelling, are discussed. An ideal quick return mechanism, properly actuated, is modelled as a test case to prove the effectiveness of the proposed approach.

AMS subject classifications: 70E55, 74H45

Key words: Flexible multibody, meshless modelling, spectral modelling, shape functions, planar flexible mechanism.

1 Introduction

Flexible multibody modelling is generally associated to a large number of degrees of freedom (dofs) and high computational costs. Therefore, flexible multi-body dynamics was often treated, in case of relatively small deflections, by means of a finite-element modelling method combined with a condensation strategy for the model, in a component mode synthesis framework [1].

The introduction of the flexibility in the floating parts of a mechanism brings in a further complexity in the non-linear system of equations that describes the evolution of motion. In [2] different approaches about flexible body modelling were discussed, leaving the classical finite element modelling (FEM) approach as a viable method to include

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flexibility. In [3,4] the contact of the wheel with the rail was modelled, but only the rail, as a non-moving part, was considered as a flexible component. Other authors [5] worked on a two-steps procedure, based on Krylov-subspaces and Gramian matrices, to obtain a reduction of number of the system equations from FEM, but the approach was not general enough, since only flexible components with no rigid body motions (fixed to the ground) were taken into account.

The modelling of flexible parts received detailed attentions in particular from the specific research community on manipulators or high speed and dexterity robots. In this niche of peculiar mechanisms, with its own conventions on transforming matrices due to the limited set of coupling pairs (or kinematical constraints, mainly rotational joints) and high number of actuators or dofs, the need of physically adherent modelling worked on increasing the accuracy by means of what Meirowitch introduced in [6], known as the eigenfunction expansion of the boundary-value problem for continuous bodies, the so-called Assumed Modes Method (AMM). In [7] an early theoretical introduction of eigenfunctions, as a truncated modal base, can be found to enhance the modelling of the flexibility description of manipulator arms, with their peculiar formulation about transformation matrices in joint coordinates. In [8] AMM was applied to simulations of flexible robots containing only rotational joints, with clamped boundary conditions at the base (hub of rotational joint) of an Euler-Bernoulli slender beam and mass boundary conditions, representing balance of moment and shearing force, at the carrying mass end of the beam. AMM was used in [9] for controlling purposes, again with clamped-mass boundary conditions and the Denavit-Hartenberg parameters, employing severe torques in the motors to have a simple input shaping able to excite, like an impulsive force, the flexible dynamics of the links. In [10] the floating frame approach, common to general multibody procedures [11,12], was paired to the AMM (declined by normalised clamped-free modes) under the small rotations assumption in view of control design and experimental comparison. Also [13] used the AMM with the clamped-free boundary conditions and the Denavit-Hartenberg parameters; instead [14] adopted AMM (clamped-free) with Lagrange multipliers for the constrained kinematics of the rotational joints and found sound agreement with FEM based approaches also when severe input torques excited strong vibrations in the links. The computational complexity and CPU time of an AMM approach was investigated in [15]. The authors of [16] and [17] simplified the 3-PRR planar flexible manipulator around the solution of the rigid-body counterpart to focus on the induced vibrations, but they adopted a pinned-pinned boundary condition for the AMM and verified it as the most close to experimental modal analysis tests. In [18] the AMM is preferred against FEM approach for its lower computational cost, with the indication that the source of errors comes from the distance of the truncated assumed mode base from the real modeshapes of the modelled beam; Lagrange motion equations are developed for flexible link manipulators and Lagrange multipliers are also introduced to account for the kinematic constraints of the rotational joints and a simulation with severe torques is commented. The flexibility of the slender links in mobile manipulators was considered by [19–22], in both FEM and AMM (pinned-pinned boundary conditions) versions, in a