Modelling and Computation for the Valuation of Two-Period $R\&D$ Projects by Option Games

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Abstract. In $R\&D$ investment projects, the value and the flexibility of investments are investigated by real option models, while the competition among the investors are mainly analyzed by the game theory. In this paper, we propose an option game model to analyze the case in which two firms invest the same project during a period of time. The firm taking the initiative, the leader, achieves more market shares, whereas the follower gains some investment information from the leader. We apply the fitted finite volume method and the power penalty method in order to get the solution of the option game model. Several managerial results including the payoffs and the effects of parameters are also discussed. Finally, by comparing the compound American option with the European one, we find the former more suitable for the analysis of $R\&D$ investment projects.

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Key words: $R\&D$ investment project, option game, compound American option, fitted finite volume method.

1 Introduction

Recently, the real option game theory is developing rapidly with the expansion of its applications in various areas. Especially, it becomes a more and more powerful tool to deal with the competition project investment problems. The fundamental analysis method of real option games consists of the real option theory, the game theory, and the traditional theory of budgets.

In the past two decades, amounts of literatures have focused on standard and non-standard real option games. The researchers study the problems about the competition

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among a few firms to invest the same project and the choice of the investment time. In particular, Smets [1] first establishes the standard of continuous time option game model, in which it is assumed that there are two companies in the competitive market. So, this kind of model is also named as the Existing Market Model. Based on Smets' research, Dixit [2] discusses the case that enterprises are not active in the market before their investments, and this type of model is named as the New Existing Market Model. In addition, Dixit [2] summarizes the Smets' model, and analyzes the case of imperfect competition circumstances. For a continuous-time framework, they use the option game method, that is, the permanent option method, to analyze the market of two competitors and incomplete information, and show the Leader and the Follower's value functions, and get the numerical solutions of their investment thresholds. Moreover, Smit [3] analyzes the aspects of competition in a discrete-time framework. Also, they combine the real option method of investment timing with the game theory, and use the standard game theory to show the influence of competition on the project value and the investment timing.

With the development of compound exchange option models, the European and American compound exchange options are studied in the articles [4–6], and [7]. Moreover, Cortelezzi [4] evaluates the *R&D* investment programs of the multiple-phase with or without overlapping, and propose to evaluate the European and the American compound exchange options through Monte Carlo simulations. Also, they present a series of numerical experiments to provide evidence for the accuracy of the proposed method. William [5] develops an evaluation equation for the European exchange option which can be executed only at the expiration date. Besides, an American exchange option, in which the underlying asset is in absence of dividends, can also be evaluated with this model. In addition, McDonald and Siegel [6] evaluate the price of a European exchange option with distributing dividends, and Carr [7] gives a valuation for an American exchange option by a special method which utilizes the pseudo-American exchange option to approximate the American exchange option.

Because of fierce competition, enterprises must secure financing or supplies before starting or winning the project. Then, it becomes an inevitable trend to combine the real option with the game theory to analyze the firms' behaviour. Nowadays, many models are available for the compound European and American exchange options, such as the ones in [8] and [9]. Furthermore, Weeds [10] considers the irreversible investment in the competing *R&D* projects with uncertain returns under a winner-takes-all patent system, and uses a standard real option framework to derive the investment thresholds for a duopoly *R&D* market. Azevedo and Paxson [8] introduce the basic aspects of the standard real option game models, and contrast the discrete-time with the continuous-time models when they apply the real option game to solve the corresponding competing projects. Giovanni [9] puts forward to the compound European exchange option model to study the interaction between two companies which invest in *R&D* projects, derives these companies' payoff functions, and computes Nash equilibriums of the two companies and the critical market values of investments.

As one of project investment evaluation methods, the real option pricing method al-