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An Immersed Boundary-Simplified Gas Kinetic Scheme for 2D Incompressible Flows with Curved and Moving Boundaries

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Abstract. In this work, we present an immersed boundary-simplified gas kinetic scheme for simulation of two-dimensional (2D) incompressible flows with curved and moving boundaries. Specifically, a fractional step technique with predictor and corrector processes is introduced to solve the governing equations. In the predictor step, the macroscopic governing differential equations are solved on the fixed Eulerian meshes by the recently developed simplified gas kinetic scheme (GKS). Compared to the conventional GKS, the simplified GKS is simpler and more efficient. At the same time, the simplified GKS inherits the advantage of good robustness of conventional GKS. In the corrector step, the velocity correction is carried out on the Lagrangian points by the implicit boundary condition-enforced immersed boundary method (IBM). Since it strictly originates from the no-slip boundary condition, this approach can avoid completely the unphysical streamline penetration phenomenon. Several numerical experiments show that the 2D incompressible flows with curved and moving boundaries can be well simulated by the developed scheme.

AMS subject classifications: 76D17, 76M12

Key words: Simplified GKS, immersed-boundary method, 2D incompressible flows, curved and moving boundaries.

1 Introduction

Flow problems with curved and moving boundaries exist widely in practical engineering, such as flows in human heart [1–3], fish swimming [4–7], insect flight [8–10] and freely falling objects [11–13]. In such problems, the large displacement and deformation

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of objects make the body-fitted mesh-based numerical methods be complicated and difficult for the application. This difficulty mainly arises from the tedious grid generation in each time step. To overcome this defect, the immersed boundary method (IBM) [14–16] has been developed and attracted increasing attention in recent years due to its simplicity and flexibility. In IBM, the effect of the solid boundary on the flow field is depicted by a restoring force and added to the momentum equation. Thus, the fluid field can be solved on the fixed Cartesian (Eulerian) meshes without considering the presence of the immersed object.

As mentioned above, there are two steps in the solution process of IBM. The first step is to solve the standard Navier-Stokes equations on the fixed Eulerian meshes. Gas kinetic scheme (GKS) [17–19] is a popular solver to do so. In this method, the Navier-Stokes equations are discretized by the finite volume method (FVM) and the numerical fluxes at the cell interface are reconstructed physically by the local solution of Boltzmann equation. Due to the strong foundation in physics, GKS performs very well in both incompressible and compressible flows [20–23]. The conventional GKS [17, 18, 20, 21] usually applies the Maxwellian function as the equilibrium state and utilizes the local integral solution of Boltzmann equation to reconstruct the numerical fluxes. Due to discontinuity of conservative variables and their derivatives at the cell interface, the local integral solution of Boltzmann equation and the final expressions of numerical fluxes are very verbose and complex as commented in the work of Tang [24]. For simulation of incompressible flows, the conventional GKS can be simplified to some extent by assuming that the flow variables and their derivatives at the cell interface are changed smoothly [25–27]. However, Chen et al. [28] recently found that this assumption may deteriorate the stability of the conventional GKS at high Reynolds numbers. So, the discontinuity of flow variables at the cell interface should be retained in order to improve the stability of incompressible GKS.

To simplify the implementation and improve the computational efficiency of the conventional GKS, a two-dimensional (2D) simplified GKS has been recently developed by Shu and his coworkers [29–31]. In the method, the Maxwellian distribution function is simplified as a circular function. Accordingly, the integrals for conservation forms of moments in the infinity domain of particle velocity space for the conventional GKS, which are needed to recover the Navier-Stokes equations, can be reduced to those in the finite domain (integrals along the circle) for the simplified GKS. Furthermore, in the simplified GKS, the numerical fluxes at the cell interface are reconstructed by the local asymptotic solution of Boltzmann equation. From the Chapman-Enskog analysis [32, 33], this local solution can be finally expressed as the linear combination of equilibrium distribution functions at the cell interface and on the circle. These simplifications make the expressions of numerical fluxes for the simplified GKS be shortened correspondingly. In addition, for simulation of incompressible flows, the computational efficiency of the simplified GKS can be further improved [34, 35]. In particular, the integral domain along the circle at cell interface can be approximately considered to be symmetric due to the incompressible limit. As a result, the formulations of numerical fluxes can be given more