Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **12**, No. 1, pp. 141-163

DOI: 10.4208/aamm.OA-2019-0126
February 2020

Superconvergence Analysis for the Maxwell's Equations in Debye Medium with a Thermal Effect

Changhui Yao[1,2,*], Dongyang Shi[1] and Mengmeng Hou[1]

[1] *School of Mathematics and Statistics, Zhengzhou University, Zhengzhou, Henan 450001, China*
[2] *State Key Laboratory of Space Weather, Chinese Academy of Sciences, Beijing 100190, China*

Received 29 April 2019; Accepted (in revised version) 18 September 2019

Abstract. In this paper, a mixed finite element method is investigated for the Maxwell's equations in Debye medium with a thermal effect. In particular, in two dimensional case, the zero order Nédélec element $(Q_{01}\times Q_{10})$, the piecewise constant space Q_0 element, and the bilinear element Q_{11} are used to approximate the electric field $\mathbf{E}$ and the polarization electric field $\mathbf{P}$, the magnetic field H, and the temperature field u, respectively. With the help of the high accuracy results, mean-value technique and interpolation postprocessing approach, the convergent rate $\mathcal{O}(\tau+h^2)$ for global superconvergence results are obtained under the time step constraint $\tau=\mathcal{O}(h^{1+\gamma})$, $\gamma>0$ by using the linearized backward *Euler* finite element discrete scheme. At last, a numerical experiment is given to verify the theoretical analysis and the validity of our method.

AMS subject classifications: 65N30, 65N15

Key words: Maxwell's equations, thermal effect, error analysis, superconvergence.

1 Introduction

In this paper, we consider the following Maxwell's equations:

$$\nabla\times\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t},\qquad \nabla\times\mathbf{H}=-\frac{\partial\mathbf{D}}{\partial t}+\mathbf{J},$$
$$\nabla\cdot\mathbf{D}=\rho,\qquad \nabla\cdot\mathbf{B}=0,$$

where $\mathbf{E}$ and $\mathbf{H}$ denote the strengths of the electric and magnetic fields, respectively. $\mathbf{D}$ and $\mathbf{B}$ are the electric and magnetic flux densities, respectively. $\mathbf{J}$ and ρ represent the

*Corresponding author.
Emails: chyao@lsec.cc.ac.cn (C. H. Yao), dy_shi@zzu.edu.cn (D. Y. Shi), 2605822173@qq.com (M. M. Hou)

current density and the density of free electric charge, respectively. The above equations will be supplemented with the constitutive laws by:

$$\mathbf{D}=\epsilon_0\mathbf{E}+\mathbf{P}, \qquad \mathbf{B}=\mu_0\mathbf{H}+\mu_0\mathbf{M},$$

where $\mathbf{P}$ and $\mathbf{M}$ represent the electric and magnetic polarization, respectively. ϵ_0 and μ_0 are the electric permittivity of free space and the magnetic permeability, respectively. We assume $\mathbf{M}=0$ since we can choose to ignore the magnetic effect among the dielectric materials.

Debye medium is one of basic physical concepts when one investigates dielectric in electromagnetic theory and materials science [22]. It is a kind of isotropic dispersive medium, and its permittivity and conductivity are functions of frequency. With the help of the polarization and dielectric relaxation, one can establish phenomenological theory in Debye medium. That is to say, in the process of polarization, microscopic particles complex energy exchange actions can be taken into the following dielectric time parameters. Therefore, numerical studies of Maxwell's equations in Debye medium have attracted considerable attention.

The linear polarization representation originates from the model proposed by Debye [3]. Similar to this representation, in this paper, we consider a linear polarization model

$$\mathbf{P}_t+\frac{1}{t_0}\mathbf{P}=\frac{\epsilon_0(\epsilon_s-\epsilon_\infty)}{t_0}\mathbf{E},$$

where ϵ_s, ϵ_∞ and t_0 stand by the static relative permittivity, the value of permittivity for an extremely high frequency field and the relaxation time of the dielectric materials, respectively.

Considering the effect of temperature field on electromagnetic field, we use Ohm's law $\mathbf{J}=\sigma(u)\mathbf{E}$ to describe the system, The unknown u is the temperature, the local density of Joule's heat generated by intensive electric waves equals [21]

$$\mathbf{E}\cdot\mathbf{J}=\mathbf{E}\cdot\sigma(u)\mathbf{E}=\sigma(u)|\mathbf{E}|^2.$$

Thus, from Fourier's law and the conservation of energy, we see that u satisfies

$$u_t-\nabla\cdot(k\nabla u)=\sigma(u)|\mathbf{E}|^2r,$$

where k is the coefficient of thermal conductivity and other physical constants such as density and specific heat have been normalized.

Throughout the paper, we suppose that σ is Lipschitz continuous with respect to u, which satisfies

$$0<\sigma_{\min}\le\sigma\le\sigma_{\max}.$$