Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **12**, No. 1, pp. 212-250

DOI: 10.4208/aamm.OA-2019-0039
February 2020

A Robust Riemann Solver for Multiple Hydro-Elastoplastic Solid Mediums

Ruo Li[1], Yanli Wang[2] and Chengbao Yao[3,*]

[1] *HEDPS & CAPT, LMAM & School of Mathematical Sciences, Peking University, Beijing, China*
[2] *College of Engineering, Peking University, Beijing, China*
[3] *School of Mathematical Sciences, Peking University, Beijing, China and Northwest Institute of Nuclear Technology, Xi'an, Shaanxi 710024, China*

Received 7 February 2019; Accepted (in revised version) 28 September 2019

Abstract. We propose a robust approximate solver for the hydro-elastoplastic solid material, a general constitutive law extensively applied in explosion and high speed impact dynamics, and provide a natural transformation between the fluid and solid in the case of phase transitions. The hydrostatic components of the solid is described by a family of general Mie-Grüneisen equation of state (EOS), while the deviatoric component includes the elastic phase, linearly hardened plastic phase and fluid phase. The approximate solver provides the interface stress and normal velocity by an iterative method. The well-posedness and convergence of our solver are proved with mild assumptions on the equations of state. The proposed solver is applied in computing the numerical flux at the phase interface for our compressible multi-medium flow simulation on Eulerian girds. Several numerical examples, including Riemann problems, shock-bubble interactions, implosions and high speed impact applications, are presented to validate the approximate solver.

AMS subject classifications: 76L05, 74H15, 76T30, 68U20, 47E05
Key words: Riemann solver, Mie-Grüneisen, hydro-elastoplastic solid, multi-medium flow.

1 Introduction

Significant interest has arisen in the modeling and simulation of dynamic events that involve high-load conditions and large deformations, such as shock-driven motions, high-speed impacts, implosions, and so on. The numerical analysis of these problems demands the implementation of very specific capabilities that enable the simulation of multiple mediums and their interactions through accurate descriptions of boundary conditions and high-resolution shock and wave capturing.

*Corresponding author.
Emails: rli@math.pku.edu.cn (R. Li), wangyanliwyl@gmail.com (Y. L. Wang), yaocheng@pku.edu.cn (C. B. Yao)

There are two typical frameworks to describe the motion of multi-medium flows [1], that is, the Lagrangian framework and the Eulerian framework. In the Lagrangian framework, the equations for mass, momentum and energy conservations are solved using a computational mesh that conforms to the material boundaries and moves with particles [2,3], which benefits from its simplicity and natural description of deformation, but suffers from mesh distortion when dealing with large deformation problems. In Eulerian framework the mesh is fixed in space, which makes these methods very suitable for flows with large deformations, such as Udaykumar et al. [4–9], Liu et al. [10–15], Mehmandoust et al. [16], Sijoy et al. [17], and so on. A typical procedure of multi-medium interaction in Eulerian grids mainly consists of two steps. The first step is the interface capture, including the diffuse interface method (DIM) [18–23], and the sharp interface method (SIM), such as the volume of fluid (VOF) method [24,25], level set method [26,27], moment of fluid (MOF) method [28–30] and front-tracking method [31,32]. The second step is the accurate prediction of the interface states, which can be used to stabilize the numerical diffusion in diffuse interface methods, and to compute the numerical flux and interface motion in sharp interface methods. One common approach is to solve a multi-medium Riemann problem which contains the fundamentally physical and mathematical properties of the governing equations and plays a key role in designing the numerical flux.

The solution of a multi-medium Riemann problem depends not only on the initial states at each side of the interface, but also on the forms of constitutive relations. There exist some difficulties in the cases of real materials due to the high nonlinearity of the equation of state and non-conservation of the deviatoric evolution. A variety of methods to solve the corresponding Riemann problems have then been proposed. For example, Yadav [33] analyzed spherical shocks in metals by employing a hydrostatic Mie-Grüneisen equation of state that does not consider the effects of shear deformation. Shyue [34] developed a Roe's approximate Riemann solver for the Mie-Grüneisen EOS with variable Grüneisen coefficient. Arienti et al. [35] applied a Roe-Glaster solver to compute the equations combining the Euler equations involving chemical reaction with the Mie-Grüneisen EOS. Lee et al. [36] developed an exact Riemann solver for the Mie-Grüneisen EOS with constant Grüneisen coefficient, where the integral terms are evaluated using an iterative Romberg algorithm. Banks [37] and Kamm [38] developed a Riemann solver for the convex Mie-Grüneisen EOS by solving a nonlinear equation for the density increment involved in the numerical integration of rarefaction curves. Unlike the fluid, there may exist more than one nonlinear wave in a solid when it undergoes an elastoplastic deformation, which will increase the difficulty to obtain the exact solution of the Riemann problem. Kaboudian et al. [39] analyzed the elastic Riemann problem in the Lagrangian framework, and established the corresponding Riemann solver according to the characteristic theory. Xiao et al. [40] raised an iterative procedure to solve the Riemann problem approximately by linearizing the Riemann invariants. Tang et al. [41] put forward a nearly exact Riemann solver for the perfectly elastoplastic solid based on the physical observation, where the Murnagham EOS and perfectly plastic model were chosen for the hydrostatic pressure and deviatoric stress respectively. Abouziarov et al. [42]