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A Viscosity-Splitting Method for the Navier-Stokes/ Darcy Problem

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Abstract. In this report, we give a viscosity splitting method for the Navier-Stokes/Darcy problem. In this method, the Navier-Stokes/Darcy equation is solved in three steps. In the first step, an explicit/ implicit formulation is used to solve the nonlinear problem. We introduce an artificial diffusion term $\theta\Delta\mathbf{u}$ in our scheme whose purpose is to enlarge the time stepping and enhance numerical stability, especially for small viscosity parameter ν, by choosing suitable parameter θ. In the second step, we solve the Stokes equation for velocity and pressure. In the third step, we solve the Darcy equation for the piezometric head in the porous media domain. We use the numerical solutions at last time level to give the interface condition to decouple the Navier-Stokes equation and the Darcy's equation. The stability analysis, under some condition $\Delta t \le k_0$, $k_0 > 0$, is given. The error estimates prove our method has an optimal convergence rates. Finally, some numerical results are presented to show the performance of our algorithm.

AMS subject classifications: 76D05, 35Q30, 65M60, 65N30

Key words: Navier-Stokes/Darcy equations, fractional step method, viscosity-splitting method, stability analysis, optimal error analysis.

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1 Introduction

The free flow coupling with the porous media flow, where the behavior of the fluid can be described by different partial differential equations in different domains, is very important in the computational fluids. In this report, we focus on the model of the Navier-Stokes equation in the surface region coupling the Darcy's law in the subsurface region [16], which is described by a mixed Navier-Stokes/Darcy's model. The first important thing is the interface conditions between the Navier-Stokes flow and the Darcy's flow. This model was firstly studied by Beavers and Joseph in [4], they gave the interface condition (named Beavers-Joseph (BJ) condition) on the interface between the surface flow and the subsurface flow. Then, this condition was simplified by Saffman [37] getting the Beavers-Joseph-Saffman (BJS) condition. This model is used in many areas, e.g., the simulation of flooding in dry areas, petroleum engineering and environmental engineering. In point view of the numerical method, how to deal with the BJS or BJ condition on the interface is a enormous challenge.

For its widely used and enormous challenge, many authors have given some great works, e.g., unified stabilized finite element formulations for the Stokes/Darcy's flow [2]; a strongly conservative finite element method for the Stokes/Darcy's flow [30]; superconvergence analysis of finite element method for the Stokes/Darcy system [1]; a posteriori error estimate for the Stokes/Darcy [15] and so on. In 2012, Layton et al. [31] split the Stokes/Darcy problems into the Stokes and Darcy problems and give four noniterative, sub-physics, uncoupling methods. The Robin-Robin domain decomposition methods for the steady-state Stokes-Darcy system with the Beavers-Joseph interface condition were shown by Cao et al. [9], Chen et al. [12] and Discacciati et al. [17]. In [23], fully-mixed finite element methods was given by Gatica et al. In [36], a decoupled finite element method for the Stokes/Darcy flow was given. In 2012, Shan, Zheng and Layton [38] presented a decoupled method using different time steps in different domains. In [41], a local discontinuous Galerkin (LDG) method for the Stokes/Darcy flow was given by Vassilev and Yotov. For the Navier-Stokes/Darcy coupling problem, several iterative methods were presented by Badea, Discacciatiaw and Quarteroni [3, 16]. The two-level method for the Navier-Stokes/Darcy problem was given by Cai et al. [7] in 2009. Girault and Riviére [25] presented a discontinuous Galerkin approximation of coupled Navier-Stokes/Darcy Equations by BJS Interface Condition. In [43], two decoupling algorithms for the steady Stokes-Darcy model based on two-grid discretizations were shown by Zhang and Yuan. In [39], we gave the decoupled modified characteristics finite element method for the time dependent Navier-Stokes/Darcy problem. The fully-mixed finite element method for Stokes-Darcy problems was given by Gatica et al. [8,24]. A decoupled preconditioning technique for a mixed Stokes-Darcy model was presented by Márquez et al. [32]. A strong coupling of finite element methods for the Stokes-Darcy problem was shown by Márquez et al. [33]. Two-grid finite element for mixed Stokes-Darcy equations was given by Hou et al. [29, 44].

The fractional step methods, which split effects due to different operators appeared in