Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **12**, No. 1, pp. 87-100

DOI: 10.4208/aamm.OA-2019-0137
February 2020

A Posteriori Error Estimates of the Galerkin Spectral Methods for Space-Time Fractional Diffusion Equations

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Received 13 May 2019; Accepted (in revised version) 5 October 2019

Abstract. In this paper, an initial boundary value problem of the space-time fractional diffusion equation is studied. Both temporal and spatial directions for this equation are discreted by the Galerkin spectral methods. And then based on the discretization scheme, reliable a posteriori error estimates for the spectral approximation are derived. Some numerical examples are presented to verify the validity and applicability of the derived a posteriori error estimator.

AMS subject classifications: 65M12, 65M70

Key words: Galerkin spectral methods, space-time fractional diffusion equations, a posteriori error estimates.

1 Introduction

Nowadays, fractional derivatives have become an important tool to describe many different types of complex mechanical and physical behaviors. Moreover, fractional calculus theory has been successfully applied in many fields such as anomalous diffusion, viscoelastic materials, geophysics and biomedical engineering. Generally speaking, fractional calculus operator is non-local, so the numerical methods which are very effective for calculating integral order differential equations may be completely invalid for fractional differential equations (FDEs). Therefore, the numerical solution of fractional differential equations has attracted more and more attention of mathematical workers [6,12, 20].

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During the last decades, there appears a growing interest in developing numerical methods for solving FDEs. And the early methods mainly include finite difference method and finite element method. For example, Deng [5] discussed a finite element method for the fractional Fokker-Planck equation and a convergence rate $\mathcal{O}(k^{2-\alpha}+h^{\mu})$ was obtained. Wang and Yang [22] studied finite element methods for variable-coefficient conservative fractional elliptic differential equations and showed that the weak formulation is well posed. Wang [21] investigated fast alternating-direction finite difference methods for three-dimensional space-fractional diffusion equations, which effectively reduced the computation and storage requirements of each iteration. Nochetto, Otárola and Salgado [19] studied the finite element approximation for parabolic equations with fractional diffusion and the stability and error estimates of the scheme were given. Jin et al. [10] analyzed Galerkin finite element methods for inhomogeneous fractional diffusion equation and the L^2- and $H^{\frac{\alpha}{2}}$-norm error estimates were derived for the semidiscrete scheme and L^2-norm error estimates were obtained for the fully discrete schemes. Zeng et al. [28] proposed a numerical method based on a fractional linear multistep methods in time and the FEM in space for time-fractional subdiffusion equation with Dirichlet boundary conditions. Bu et al. [1] investigated finite difference/finite element method for two-dimensional space and time fractional Bloch-Torrey equations, the stability and convergence of the semidiscrete scheme and fully discrete scheme were proved. Hou, Tang and Yang [8] considered the fully discretized Crank–Nicolson scheme for fractional-in-space Allen-Cahn equations and showed that the numerical solutions satisfy discrete maximum principle under reasonable time step constraint. Recently, there are many works for the fractional differential equations [11,26]. For example, Yue et al. [27] considered a fully finite element adaptive adaptive algebraic multigrid (AMG) method for time-space Caputo-Riesz fractional diffusion equations, which have the well robustness and high efficiency compared with the classical AMG method. Xing and Wen [23] considered a class of two-dimensional Riesz space-fractional diffusion equations by the alternating direction implicit Crank-Nicholson (ADI-CN) method, which reduces the computational complexity and is unconditionally stable. Gunzburger and Wang [7] studied the time fractional partial differential equation by the Crank-Nicolson method, which achieves second-order convergence in time under the regularity assumptions of the source and initial data.

Fractional differential equations have non-local operators, which will inevitably lead to the overall dependence of numerical solutions, i.e., the full algebraic system. So the advantage of sparsity for the low-order method over the high-order method cannot be reflected. As a global high-precision algorithm, spectral methods have apparent superiority and become the preferred algorithm to solve this kind of equation. Several spectral methods for FDEs have been proposed recently, for instance Lin and Xu [16] proposed a numerical method based on a finite difference scheme in time and Legendre spectral method in space for time fractional diffusion equation. They also proposed a space-time spectral method for the time fractional diffusion equation and derived a priori error estimate [13]. Chen, Shen and Wang [4] considered the General Jacobi functions Petrov-