Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **12**, No. 2, pp. 301-318

DOI: 10.4208/aamm.OA-2018-0207
April 2020

Efficient Preconditioned Iterative Linear Solvers for 3-D Magnetostatic Problems Using Edge Elements

Xianming Gu[1], Yanpu Zhao[2,*], Tingzhu Huang[3] and Ran Zhao[4]

[1] *School of Economic Mathematics/Institute of Mathematics, Southwestern University of Finance and Economics, Chengdu 611130, Sichuan, China*
[2] *School of Electrical Engineering and Automation, Wuhan University, Wuhan 430072, Hubei, China*
[3] *School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu 611731, Sichuan, China*
[4] *Key Laboratory of Intelligent Computing and Signal Processing, Ministry of Education, Anhui University, Hefei 230039, Anhui, China*

Received 27 September 2018; Accepted (in revised version) 11 October 2019

Abstract. For numerical computation of three-dimensional (3-D) large-scale magnetostatic problems, iterative solver is preferable since a huge amount of memory is needed in case of using sparse direct solvers. In this paper, a recently proposed Coulomb-gauged magnetic vector potential (MVP) formulation for magnetostatic problems is adopted for finite element discretization using edge elements, where the resultant linear system is symmetric but ill-conditioned. To solve such linear systems efficiently, we exploit iterative Krylov subspace solvers by constructing three novel block preconditioners, which are derived from conventional block Jacobi, Gauss-Seidel and constraint preconditioners. Spectral properties and practical implementation details of the proposed preconditioners are also discussed. Then, numerical examples of practical simulations are presented to illustrate the efficiency and accuracy of the proposed methods.

AMS subject classifications: 65F08, 65N30, 78A30

Key words: Coulomb gauge, edge element, iterative linear solver, magnetostatics, block preconditioner.

---

1 Introduction

Three-dimensional (3-D) magnetostatic field computations are widely solved for the local magnetic flux densities [6–8,12] or global inductance parameters [9] of devices excited

*Corresponding author.
Emails: guxianming@live.cn (X.-M. Gu), yanpu.zhao@whu.edu.cn (Y. P. Zhao), tingzhuhuang@126.com (T.-Z. Huang), rzhao@ahu.edu.cn (R. Zhao)

by direct currents (DC) and/or permanent magnets (PM). In practical applications, the solution domain usually contains arbitrarily-shaped objects having piecewise or nonlinear permeability coefficients, so the finite element method (FEM) is a good and versatile choice and produce accurate numerical solutions. There are mainly three ways to formulate magnetostatic problems for numerical computation, including the field formulations [8–10], magnetic scalar potential (MSP) formulations [11, 15] and magnetic vector potential (MVP) formulations [7, 12, 14–16]. Among these formulations, the MVP-based formulations are the most convenient in implementation and thus widely in use. Tedious pre-processing of the source currents for multiply-connected conductors is not necessary when using gauged MVP formulations [9]. What is more, the MVP actually has independent physical reality instead of generally a useful mathematical artifice [13]. Thus it is really necessary and important to study gauged MVP formulations, such as the Coulomb gauged ones [7, 12, 14], to solve the MVP which has a unique solution now.

For spatial discretization of the MVP using FEM, since nodal finite elements impose both tangential and normal continuity when used to approximate each component of the MVP $\vec{A}$, which physically only has tangential continuity across material interfaces, it can produce large numerical errors at material interfaces of iron and air or reentrant corners of geometry objects [15]. Alternatively, edge elements have become popular in computational magnetics [7, 10, 12] because of their built-in property by allowing proper discontinuity of the normal components of the MVP, which can be also widely found in eddy-current problems [1, 10] or coupled inductive-capacitive problems [4, 5].

The Coulomb-gauged formulation using penalty technique can produce a linear system with a unique solution [14], but this is only useful for nodal elements and not valid for edge elements since the divergence values of the edge element basis is zero within each mesh element [16]. In this paper, the MVP formulation proposed in [7] is adopted to model the physical problem being solved, where edge elements can still be used for spatial discretization while satisfying the Coulomb gauge. After finite element spatial discretization, it will result in large-scale sparse systems of linear equations when fine mesh is involved in computation to ensure sufficient numerical accuracy. Due to the high demand of memory, direct solvers are usually too expensive to use and impractical. Thus there is a strong need for efficient and robust solvers of the resultant linear systems.

For large-scale applications, although sparse direct solvers are generally accurate, robust and predictable in terms of both storage and computational cost, they tend to be too expensive to use for solving large-scale linear systems especially in terms of memory (e.g., see [17–19]). Iterative solvers, namely the well-known class of Krylov subspace methods (KSMs), can be an attractive alternative to sparse direct methods as they only require the information of matrix-vector products; see e.g., [21–23] and references therein. Since the resultant system of the proposed formulation is ill-conditioned, so the convergence of the KSM will become very slow [21, 24]. To remedy this dilemma, we also establish two novel block preconditioners for accelerating KSMs to solve the linear system. Moreover, practical implementation of our preconditioned KSMs (PKSMs) will be discussed in details.