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Two-Grid Stabilized FEMs Based on Newton Type Linearization for the Steady-State Natural Convection Problem

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Abstract. This paper is concerned with two types of two-grid stabilized finite element methods (FEMs) based on Newton iteration for the steady-state nature convection problem. The first method needs to solve one small nonlinear natural convection system on the coarse mesh with mesh width H, and then to solve one large linearized natural convection system on the fine mesh with mesh width $h=\mathcal{O}(H^2)$ based on Newton iteration. The other method needs to solve one small nonlinear natural convection system on the same coarse mesh, and then to solve two large linearized systems on the fine mesh with mesh width $h=\mathcal{O}(H^{\frac{7-\varepsilon}{2}})$ based on Newton iteration which have the same stiffness matrix with only different right-hand side. In both methods, the stabilization terms are defined via two local Gauss integrations at element level which has no need to introduce additional variables comparing with the standard variational multiscale stabilized FEMs. The stability estimates and the convergence analysis for both methods are derived strictly. Ample numerical results are presented to confirm the theoretical predictions and demonstrate the efficiency of the new methods.

AMS subject classifications: 65N15, 65N30, 65N12

Key words: Natural convection problem, two-grid method, finite element methods, Newton iteration, variational multiscale stabilization, error estimates.

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1 Introduction

In this paper, we consider the numerical approximation of the steady-state incompressible nature convection problem, whose dimensionless form including solid media is given

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by

$$-Pr\Delta u+(u\cdot\nabla)u+\nabla p=PrRaTe+b_1 \qquad \text{in } \Omega_f, \tag{1.1a}$$

$$\nabla\cdot u=0 \qquad \text{in } \Omega_f, \tag{1.1b}$$

$$u=0 \qquad \text{on } \partial\Omega_f, \qquad u\equiv 0 \qquad \text{in } \Omega-\Omega_f=\Omega_s, \tag{1.1c}$$

$$-\nabla\cdot(k\nabla T)+(u\cdot\nabla)T=b_2 \qquad \text{in } \Omega, \tag{1.1d}$$

$$T=0 \qquad \text{on } \Gamma_T, \qquad \frac{\partial T}{\partial n}=0 \qquad \text{on } \Gamma_B, \tag{1.1e}$$

where Ω is an open bounded domain in $\mathbb{R}^d$ $(d=2,3)$, the unknowns are the velocity field u, the pressure p and the temperature T, e is a unit vector in the direction of gravitational acceleration, b_1 and b_2 are given functions, n is the outward unit normal to Ω. Here Ω_s, Ω_f are disjoint polygonal or polyhedral domains, and $\Gamma_T=\partial\Omega\backslash\Gamma_B$, where Γ_B is a regular open subset of $\partial\Omega$. The positive parameters Pr, Ra and k denote the Prandtl number, the Rayleigh number and the thermal conductivity, respectively. Moreover, $k=k_f$ in Ω_f and $k=k_s$ in Ω_s, where k_f and k_s are positive constants which denote the thermal conductivity for the different media. The system (1.1) uses Boussinesq approximation as governing equations.

Natural convection problem can describe many phenomena in industrial application, such as room ventilation, katabatic winds, solar collectors, dense gas dispersion, cooling of electronic equipments and nuclear reactors, double glass window design, etc. Even, it can explain some rules of the ocean and atmospheric dynamics. Therefore, this problem is very valuable in our real life and it is of practical interest to design efficient numerical simulation methods for natural convection problem. There are many works devoted to the development of the efficient numerical methods for natural convection problem, we can see the famous benchmark solutions of de Vahl Davis [9], which used second-order central approximations to solve natural convection problem in a square cavity. A semi-implicit form of the characteristic-based split scheme for natural convection problem was developed by Massarotti [30]. An explicit finite element algorithm for natural convection problem was employed by Manzari [29]. Wan et al. [36] used discrete singular convolution method for the solution of the natural convection problem. Boland and Layton [3,4] presented finite element method (FEM) for solving stationary natural convection problem and established the theoretical framework of FEM for solving it.

As we know, if we solve natural convection problem by the standard Galerkin method, it may exhibit global spurious oscillations [11,31] and yield inaccurate approximation. One reason is the dominance of the convection term. There are numerous works devoted to the development of efficient schemes to deal with such problem. Among them, we list some recently developed stabilized methods as follows. The variational multiscale (VMS) method is based on the decomposition of the flow scales. In a type of VMS method, the flow are decomposed into the large scales and small scales, and the former are defined by projection into appropriate subspaces. For more details, we can see [12,16,17,19,20] and the references therein. Li and He [26] first proposed a stabilized finite element method