Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **12**, No. 2, pp. 503-526

DOI: 10.4208/aamm.OA-2019-0011
April 2020

Numerical Assessment of Criteria for Mesh Adaptation in the Finite Volume Solution of Shallow Water Equations

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Received 14 January 2019; Accepted (in revised version) 22 May 2019

Abstract. We present a numerical assessment of a class of criteria for mesh adaptation in the finite volume solution of shallow water flows. The shallow water equations are numerically approximated by a predictor-corrector procedure in unstructured triangular meshes. The numerical fluxes at the interfaces of each triangle are reconstructed in the predictor stage using an upwind scheme along with slope limiters to achieve a second-order accuracy. Treatment of source terms is performed in the corrector stage using a well-balanced technique. Four error indicators using the flow variables are discussed and applied as criteria for the mesh adaptation. Numerical results are presented for two test examples for a circular dam-break flow and dam-break problem over a single building. The presented criteria are found to give accurate results in comparison with similar simulations carried out using uniformly refined fixed meshes. Dynamic grid adaptation and the use of an explicit time integration scheme are found to enhance the computational efficiency of the finite volume solution of shallow water flows. In addition, the obtained results for dam-break problems are considered to be representative, and might be helpful for a fair rating of criteria for mesh adaptation in the finite volume solution of shallow water flows, particularly in long time computations.

AMS subject classifications: 65M08, 35L53, 76B15, 74J40, 76B07

Key words: Shallow water equations, finite volume methods, mesh adaptation, unstructured grids, dam-break problems.

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1 Introduction

For the past years, finite volume methods have been widely used for the numerical solution of shallow water equations in both one and two space dimensions. Authors in [12] have modified the well-known Roe method for hyperbolic systems of conservation laws [29] to solve shallow water equations. This method has also been applied for one-dimensional channel flows in [34]. However, this type of finite volume methods tend to become computationally demanding for practical applications due to their treatment of source terms. A class of well-balanced finite volume methods have been proposed in [18] for shallow water equations with cross-section irregularities. Solving two-dimensional shallow water equations using the flux difference splitting has been studied in [20] among others. Solving shallow water equations using Godunov-type finite volume methods have been studied in [2] whereas, exact solutions for the Riemann problem at the interface with a sudden variation in the topography have been presented in [1]. In [23], a finite volume method is applied to shallow water equations using quasi-steady wave propagation techniques. This method solves an additional Riemann problem at the center of each cell to balance the discretizations of source terms and flux gradients. However, the extension of this finite volume method for unstructured meshes is not trivial. Finite volume methods based on the local hydrostatic reconstruction have been investigated in [4] for open channel flows and the extension of these methods to high-order accuracy was proposed in [26]. It consists of evaluating the cell-boundary values of the free-surface elevation and the water depth coupled with a correction of the source term quadrature in these methods. Using the surface elevation instead of the water height in the shallow water equations, central-upwind finite volume methods have been studied in [21] among others. Application of Essentially Non-Oscillatory (ENO) and Weighted Essentially Non-Oscillatory (WENO) finite volume schemes to shallow water equations has been discussed for example in [35]. However, most ENO and WENO schemes that solves real flows correctly are still very computationally demanding. Finite volume methods using kinetic discretizations have been proposed in [27] and relaxation-based finite volume methods have been applied to shallow water equations in [31], but the complexity of this class of method is relevant.

In the present study, we consider a finite volume non-homogeneous Riemann solver for the numerical solution of shallow water equations on unstructured triangular grids. The formulation and analysis of this method can be found in [30]. In particular, the discretization of the gradient fluxes using the sign matrix of the Jacobian is described in details for hyperbolic systems of conservation laws with source terms. In [9, 10], the implementation of the present finite volume method on unstructured grids is analyzed and applied to shallow water flows. This implementation involves an original treatment of the flux derivatives coupled with the source term in unstructured meshes. The method is highly accurate and well-balanced with the ability to handle simulations of slowly varying flows as well as rapidly varying flows in irregular domains with complex topographies. The current work improves the finite volume method proposed and studied