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# A Modulus Iteration Method for SPSD Linear Complementarity Problem Arising in Image Retinex

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**Abstract.** Retinex theory explains that the image intensity is the product of the object's reflectance and illumination. However, the true color of the object in the image is determined only by the reflectance of the object. The purpose of retinex problem is to decompose the reflectance from the image intensity. In this paper, a new variational model with physical constraint imposed on the reflectance is proposed. The proposed model can be transformed to a linear complementarity problem (LCP) with symmetric positive semi-definite (SPSD) matrix. The main contribution of the paper is that the LCP with SPSD matrix is solved by the modulus iteration method and the convergence is demonstrated. Experiments numerically show the effectiveness of the proposed method for retinex problem and the convergence of the modulus iteration method for solving the LCP with SPSD matrix.

**AMS subject classifications**: 65Y99, 68U10

**Key words**: Linear complementarity problem, modulus iteration method, image retinex, symmetric positive semi-definite.

## 1 Introduction

The retinex theory was proposed by Land and McCann in [1] to explain how the human visual system perceives the colors of objects. The establishment of such theory was based on optical experiments. They found in experiments, though the visible light $S$ reaching human eyes is the product of the object's reflectance $R$ $(0<R\leq 1)$ and illumination $L$ $(L>0)$ as

$$S=R\circ L, \tag{1.1}$$

the object's color perceived by human eyes is only dependent on the intrinsic reflectance of the object and irrelevant to the illumination. That is, both human eyes and brain are involved in the perception of objects' colors. Which is the reason why the theory is named

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as retinex, a compound word of retina and cortex. In (1.1), $\circ$ denotes the Hadamard product. However, the intensities of images taken by cameras are totally determined by visible light reaching the camera lens. The reflectance of an object is fixed, so the object's color in the recorded image varies with varied illuminations. The colors of an object are usually different in images when taken under different illumination conditions. When illumination is lower, the color may be darker, while the color may be brighter when the illumination is higher. In imaging applications, we undoubtedly need to try to obtain the images that can reveal the true colors of objects. The aim of retinex problem is to decompose $R$ and $L$ from the recorded image so that we can get the intrinsic reflectance and thus get true colors of the objects. Retinex problem has been applied in a lot of imaging application areas, for example, image enhancement [2–5], image editing [6], shadow removal [7], high dynamic range image tone mapping [8, 9], remote sensing image correction [10, 11], target selection and tracking [12, 13] etc.

Many methods for retinex problem have been proposed in the literature. These methods include the path-based algorithms such as those proposed in [1, 14], the kernel-based methods in [15], the PDE-based algorithms in [16–19], the center-surround retinex algorithms in [5, 20–22] and variational methods in [7, 10, 11, 23–29]. The variational methods are an important kind of methods developed in recent years. The variational framework is actually a minimization problem with respect to either the illumination or reflectance variable or sometimes with respect to both variables. The first variational method for retinex problem was proposed by Kimmel et al. in [29]. The cost function in Kimmel's method was constructed based on the assumption that illumination is spatially smooth and the recovered reflectance is obtained by division of the observed intensity over the estimated illumination. The spatial smoothness has been imposed on illumination in the construction of almost all variational methods. In [26], by also imposing the spatial smoothness assumption on the reflectance, a cost function with respect to both reflectance and illumination was constructed and both variables were recovered simultaneously. In methods such as the one proposed in [25], reflectance was assumed to have sharp changes and total variational regularization was applied for the reflectance in the cost function. However, as what pointed out in [30], total variation-regularized model can be efficient to recover piecewise constant images due to its advantage in recovering image edges, but it often leads to information loss in reflectance recovery.

The value of the reflectance $R$ should satisfy the constraint $0 < R \leq 1$ and thus $L \geq S$. Most of the variational methods met such requirement by using projection steps in iterations of the algorithms. Only a few methods fulfilled such constraint in the construction of the models for retinex problem, for example, the ones proposed in [26, 30] and [31]. Particularly, the model constructed in [31] was proved to be equivalent to a linear complementarity problem (LCP) with its matrix being symmetric positive definite and the modulus iteration method was applied to solve it. In this paper, by assuming spatial smoothness priors on both reflectance and illumination, a new variational method with physical constraints imposed on reflectance values for retinex problem is constructed. In the proposed model, the two auxiliary terms used in the cost function of the method pro-