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High-Order Three-Scale Computational Method for Thermoelastic Behavior Analysis of Axisymmetric Composite Structures with Multiple Spatial Scales

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Abstract. This study develops a novel high-order three-scale (HOTS) computational method to accurately simulate and analyze the thermoelastic behaviors of axisymmetric composite structures with multiple spatial scales. The inhomogeneities in composite structures are taken into account by periodic distributions of representative unit cells on the mesoscale and microscale. Firstly, the multiscale asymptotic analysis for these multiscale problems is given in detail, and the new unified micro-meso-macro HOTS approximate solutions for these multiscale problems are established based on the above-mentioned multiscale analysis. Two types of auxiliary cell functions are established on mesoscale and microscale. Also, two kinds of equivalent material parameters are calculated by up-scaling procedure on the mesoscale and microscale, and the homogenized problems are subsequently defined on global structure. Then, the numerical accuracy analyses for the conventional two-scale solutions, low-order three-scale (LOTS) solutions and HOTS solutions are obtained in the pointwise sense. By the foregoing error analyses, the vital necessity of developing HOTS solutions for simulating these three-scale problems is illustrated clearly. Furthermore, the corresponding HOTS numerical algorithm based on finite element method (FEM) is brought forward in detail. Finally, some numerical examples are presented to verify the usability and effectiveness of the HOTS computational method developed in this work.

AMS subject classifications: 74S05, 35J57

Key words: Multiscale asymptotic analysis, thermoelastic problems, axisymmetric composite structures, multiple spatial scales, HOTS numerical algorithm.

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1 Introduction

With the rapid development of modern material science, many natural and synthetic composite materials with more than two spatial scales have been designed and manufactured for engineering application, such as hierarchical layered materials and concrete materials, etc. The advantageous properties of these composites are remarkable due to the hierarchical and functional relationships between each of the scales. Owing to their excellent thermal and mechanical properties, these composite materials have been widely applied in the fields of aeronautic and aerospace engineering, which are usually served under complex thermo-mechanical environments. The use of these composite materials requires understanding the variations in the field variables, such as temperature, heat flux, displacement, strain and stress, both at micro-, meso- and macro-scales. In order to gain the optimal design for engineering structures, it is of great significance to accurately simulate and analyze the thermo-mechanical coupled behaviors of these composite materials. Hence, the accurate evaluation of the thermal and mechanical behaviors of these composites has attracted lots of attention from scientists and engineers.

In recent years, numerical modeling becomes a powerful tool for predicting the effective behavior of heterogeneous materials with the rapid development of computer science and technology. Generally, the equations, which govern the coupled heat conduction and mechanical deformation processes for composite materials with multiple spatial scales, have rapidly varying coefficients. The rapidly oscillating coefficients in these thermoelastic equations are applied to describe the multiscale thermoelastic behaviors of composite materials with multiple spatial scales. The solutions of temperature and displacement fields of this multiscale governing equations can exhibit large variations and even discontinuities. In the past few decades, mathematicians and engineers have developed various multiscale methods to study the multiscale behaviors of the composite materials, including the asymptotic homogenization method (AHM), variational multiscale method (VMS), heterogeneous multiscale method (HMM) and multiscale finite element method (MsFEM) [5–7], etc. Among them, the AHM is extensively applied in actual engineering applications because it has a rigorous mathematical foundation and can combine with FEM very well. For the sake of improving the inadequate numerical accuracy of conventional homogenization method, Cui et al. systematically developed a second-order two-scale (SOTS) method to accurately predict the physical and mechanical behaviors of composites, which offers a feasible framework of two-scale computation for the composite materials [8–13].

To the best of our knowledge, some studies have been performed on the composite materials with more than two spatial scales. But their studies only discussed the composites with multiple periodic structures in cartesian coordinates [14–25]. In addition, most of them simply focus on predicting and analyzing the effective material parameters of these composite materials [15–18, 21–25]. The multiscale methods they adopted also can not completely decouple the scale coupling between microscale and mesoscale, which is caused by the unreasonable assumption meso-coordinates $\mathbf{y}=\frac{\mathbf{x}}{\varepsilon}$ and micro-coordinates