Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **12**, No. 3, pp. 774-796

DOI: 10.4208/aamm.OA-2019-0054
June 2020

A Type of Cascadic Adaptive Finite Element Method for Eigenvalue Problem

Fei Xu[1], Qiumei Huang[1], Shuangshuang Chen[1] and Hongkun Ma[2,*]

[1] *Beijing Institute for Scientific and Engineering Computing, College of Applied Sciences, Beijing University of Technology, Beijing 100124, China*
[2] *Sun Yat-Sen Business School, Sun Yat-Sen University, Guangzhou 510275, Guangdong, China and Zhuhai Financial Investment Group, Zhuhai 519031, Guangdong, China*

Received 27 February 2019; Accepted (in revised version) 17 December 2019

Abstract. In this paper, a type of cascadic adaptive finite element method is proposed for eigenvalue problem based on the complementary approach. In this new scheme, instead of solving the eigenvalue problem in each adaptive finite element space directly, we only need to do some smoothing steps for a boundary value problems on each adaptive space and solve some eigenvalue problems on a low dimensional space. Hence the efficiency can be improved since we do not need to solve the eigenvalue problems on each adaptive space which is time-consuming. Further, the complementary error estimate for eigenvalue problem will be introduced. This estimate can not only provide an accurate error estimate for eigenvalue problem but also provide the way to refine mesh and control the number of smoothing steps for the cascadic adaptive algorithm. Some numerical examples are presented to validate the efficiency of the proposed algorithm in this paper.

AMS subject classifications: 65N30, 35J61, 65M55, 65B99

Key words: Adaptive finite element method, cascadic multigrid method, eigenvalue problem, complementary method.

1 Introduction

This paper is to introduce a type of cascadic adaptive finite element method for eigenvalue problem since eigenvalue problem plays an important role in large scale scientific and engineering computing. In order to design an efficient algorithm, we resort to the multigrid methods. As we know, multigrid methods [7–9, 20, 33, 41, 43] provide optimal

*Corresponding author.
Emails: xufei@lsec.cc.ac.cn (F. Xu), qmhuang@bjut.edu.cn (Q. Huang), chenshuangshuang@bjut.edu.cn (S. Chen), mahongk@mail.sysu.edu.cn (H. Ma)

algorithms for elliptic boundary value problem. The error bounds of the approximate solutions obtained from these efficient numerical algorithms are comparable to the theoretical bounds determined by the finite element discretization, while the amount of computational work involved is proportional only to the number of unknowns in the discrete problems. There exist many kinds of multigrid method such as full multigrid method and cascadic multigrid method, etc. All these different methods have their own merits. In this paper, we adopt the cascadic multigrid method [6] due to its efficiency and simplicity. The difference between cascadic multigrid method and other multigrid methods lies in the correction step. The cascadic method goes from the coarsest mesh to the finest mesh directly and never goes back to the previous level. In each level of the multigrid finite element spaces, we only need to do some smoothing steps by a conventional iteration method and the approximate solution obtained from the previous level acts as the initial value. However, we know the error obtained from the coarse grid is of relatively low frequencies when the iteration reaches the fine grid, which is very hard to eliminate by using the conventional smoother. That's why the correction step is needed in usual multigrid method. To overcome this problem and reach the desired accuracy, the cascadic multigrid method has to solve the partial differential equation on each level to the same accuracy as the requirement of the final level. In cascadic multigrid algorithm, this target is achieved by increasing the number of smoothing steps on coarser levels. Fortunately, the optimality of computational work of cascadic algorithm can still be held due to the low dimensions of the coarser levels. Besides, the error of the final approximate solution is of the same order as the discretization error of the standard finite element method if the number of smoothing steps is proportional to the number of unknowns on the final level. The cascadic multigrid method applied to eigenvalue problem can be found in [21]. For more information about the cascadic multigrid method, please refer to [6, 34, 35, 39] and the references cited therein.

The second technique will be used in this paper is adaptive finite element method (AFEM). Since the concept of AFEM was proposed by Babuška and his collaborators in [3,4], AFEM has been widely used to solve partial differential equations with singularities and the efficiency of AFEM has already been demonstrated theoretically and numerically. The theoretical analysis of AFEM is well-developed. For instance, Dörfler [19] introduced Dörfler's marking strategy and proved strict energy error reduction for the Laplace problem provided the initial mesh is fine enough. Morin, Nochetto, and Siebert [28] proved that there is no strict energy error reduction in general by introducing the concept of interior node property and data oscillation. Mekchay and Nochetto [27] gave a similar result for general second order elliptic operators by introducing the concept of total error. For eigenvalue problems, the AFEM is also a competitive candidate and the similar results can be found in [14, 17, 22], etc. For more results about AFEM, please refer to [13, 36] and the references cited therein.

Recently, a type of multilevel correction method with optimal efficiency for eigenvalue problems has been proposed in [21,25,26,40]. The purpose of this paper is to study the cascadic adaptive finite element method for eigenvalue problems based on the adap-