Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **12**, No. 4, pp. 1035-1056

DOI: 10.4208/aamm.OA-2019-0100
August 2020

Optimization and Identification of the Shape in Elastoplastic Boundary Problems Using Parametric Integral Equation System (PIES)

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Received 9 April 2019; Accepted (in revised version) 20 December 2019

Abstract. The paper presents optimization and identification of the shape of elastoplastic structures. The optimization process is performed by the particle swarm method (PSO), while direct boundary value problems are solved using the parametric integral equation system (PIES). Modeling the boundary and the plastic zone in PIES is done globally by the small number of control points of parametric curves and surfaces. Such way of defining is very beneficial in comparison to so-called element methods (finite or boundary), because it reduces the number of design variables and does not enforce re-discretization during each shape change. Together with advantages of PSO it is an effective approach to solving optimization problems. There are three examples in the paper: two of identification of the shape and one in which an optimal shape is searched.

AMS subject classifications: 65M32, 65M38

Key words: Identification, optimization, plasticity, nonlinear, parametric integral equation system (PIES), particle swarm optimization (PSO).

1 Introduction

Shape optimization (or identification) is one of the stages of the design of mechanical structures. The solution to these problems is usually achieved by minimization of the objective function, which describes the optimization criterion. In practice, it leads to multiple solving of direct boundary problems with the modified geometry. Therefore, it is extremely important, especially in the case of elastoplastic problems, to apply the appropriate method for solving direct problems. Most popular are the finite element method (FEM) [1–3] and the boundary element method (BEM) [2,4,5], however both of them have

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a serious disadvantage–they require discretization of the boundary and the domain (at least the plastic zone). Taking into account that the shape is constantly changing during optimization process, such discretization is performed many times. Each modification leads to changing positions of many nodes of boundary elements and cells. Moreover, in FEM and BEM the mentioned mesh of elements is also required for approximation of solutions. Hence its form is determined not only by the shape, but also by the boundary conditions and the expected accuracy of solutions. In the literature are some attempts to modelling the optimized boundary by parametric curves [6, 7] also in FEM and BEM. It significantly reduces the number of design variables, but the numerical solution of the direct problem still requires division into elements.

Considering the above, the parametric integral equation system (PIES) [8, 9] is proposed for solving direct elastoplastic problems. It is characterized by the separation of shape approximation from solutions approximation. It allows for global, elementless modeling of the boundary (by curves) and the domain (by surface patches) even in direct problems and independent global approximation of solutions (by approximation series). Curves and surfaces can be easily defined and then modified by the small number of control points [10, 11]. The plastic zone in PIES is defined only once by the single surface and then is only changed (by some control points) during optimization process. Changes in the surface are performed automatically with changes of the boundary, represented by curves, using overlapping control points. Thus, PIES effectiveness stems from a reduced number of design variables (only control points of curves), the lack of discretization and automatic adaptation of its mathematical formalism to the changing shape. The latter is due to PIES's main feature–the shape is analytically integrated in its mathematical formula by expressions which describe curves and surfaces.

For optimization purposes the particle swarm optimization (PSO) method [12, 13] is applied. As indicated in recent papers it is more effective than classical evolutionary algorithms, because allows for greater diversity and exploration over a single population with lower computational cost. This is especially important in elastoplastic problems solved by an incremental iterative scheme, because in inverse problems we have to deal with nested iterative processes that significantly increase the computational effort. However, it should be mentioned that the PIES method gives the possibility of applying also other optimization methods.

The proposed strategy has been tested on several examples. Shape identification has been performed basing on values from selected measurement points on the boundary. During shape optimization, minimizing of plastic deformation areas has been conducted. Obtained results confirm the efficiency of the proposed method.

2 Parametric integral equation system (PIES) for elastoplastic boundary value problems

The parametric integral equation system (PIES) for elastoplasticity in the initial strain for-