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A Nonconforming Nitsche's Extended Finite Element Method for Elliptic Interface Problems

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Abstract. This article proposes a new P_1 nonconforming Nitsche's extended finite element method for elliptic interface problems with interface-unfitted meshes. It is shown that the stability of the discrete formulation is independent of not only the mesh size and the diffusion parameters, but also the position of the interface, showing a robustness over the location of interface. In spite of the low regularity of interface problems, the optimal convergence is obtained. Numerical experiments are carried out to validate theoretical results.

AMS subject classifications: 65N30, 65N12

Key words: Nonconforming extended finite element, Nitsche's method, elliptic interface problems, interface-unfitted mesh.

1 Introduction

Interface problems are often encountered in scientific calculations, which are produced by the interaction of multi-mediums and multi-physical fields, such as electromagnetic wave propagations, fluid mechanics, biological sciences and materials sciences. There has been a growing interest in interface problems in the past four decades, and a vast of literature is available, see [4, 6, 8, 12, 14–16, 21–25, 27, 28] and so on, just to name a few. When simulating such an interface problem, discontinuities can occur anywhere, which poses challenges to the solving process.

There are two major classes of finite element methods (FEM) for solving interface problems, namely interface-fitted FEM and interface-unfitted FEM, according to whether the mesh is fitted to the interface or not. However, it is usually a nontrivial and time-consuming task to construct good interface-fitted meshes for the problems involving geometrically complicated interfaces. Moreover, remeshing is required when the interface

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evolves with time. Thus interface-unfitted methods have become highly attractive. One valid type of interface-unfitted methods is the extended finite element method (XFEM). XFEM was originally introduced by Belytschko et al. in [4] to solve elastic crack problems. A combination of Nitsche's method and the extended finite element method (NXFEM) was originally proposed in [22]. One main feature of NXFEM is that the basis functions can be discontinuous when across the interface. Its idea is to enforce the jump conditions weakly using a variant of Nitsche's method on the interface. There are lots of works about the NXFEM, see [6, 12, 14–16, 23–25, 28].

It is well-known that nonconforming finite element methods have many advantages, especially for elasticity problems and Stokes problems. In [14], Capatina et al. first proposed a combination of Nitsche's method and the P_1 nonconforming extended finite element method. Under the assumption that there is no small sub-element creating when interface elements are cut by the interface (see [14, (7)]), by adding some stabilization terms on the cut edges to the weak form, the robust stability result and optimal a priori error estimates are derived, which are independent of the mesh size, the diffusion parameters, and the position of the interface. Later Capatina et al. in [16] proposed two extensions for P_1 nonconforming case by modifying stabilization terms or the basis functions on the cut cells. Under another assumption (see [16, (9)]), similar results were obtained.

The main contribution of the current work is to remove above assumptions in [14] and [16]. In this paper, we develop a new P_1 nonconforming Nitsche's extended finite element method (Nonconforming-NXFEM) to solve elliptic interface problems. By adding several stabilization terms defined on the transmission edges to the weak form, the robust stability result and optimal a priori error estimates are obtained, which are uniform with respect to the mesh size, the diffusion parameters and the position of the interface without other assumption. Even if very small sub-elements are created when the interface cuts an element, our results are still robust. Besides, we only need one fixed constant as the weight of means in the bilinear form, however, the weights used in [14] and [16] are dependent on the diffusion parameters and the mesh-interface geometry.

The outline of this article is as follows. In Section 2 we introduce the elliptic interface problem and its discrete formulation. Section 3 presents the stability of the discrete problem. The error estimate in H^1-norm is derived in Section 4. Section 5 carries out some numerical examples to validate theoretical results. Finally, conclusions are provided in Section 6. Throughout this paper, we will use c and C with or without subscripts to denote generic positive constants, which are always independent of the mesh size, the diffusion parameters and the position of the interface.

2 Preliminaries

2.1 The elliptic interface problem

We consider the following elliptic interface problem in a convex polygonal Ω in $\mathbb{R}^2$, with a $\mathcal{C}^2$-continuous interface Γ dividing Ω into two open sub-domains Ω_1 and Ω_2 (see Fig. 1