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A Localized Space-Time Method of Fundamental Solutions for Diffusion and Convection-Diffusion Problems

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Abstract. In this paper, a localized space-time method of fundamental solutions (LSTMFS) is proposed to solve the diffusion and convection-diffusion problems. The proposed LSTMFS only requires some arbitrarily-distributed nodes inside the space-time domain and along its boundary. The local subdomain corresponding to each node can firstly be determined based on the Euclidean distance between the nodes. Then, the variable at each node can be expressed as a linear combination of variables at its supporting nodes. By solving a resultant sparse system, the variable at any node in the considered space-time domain can be obtained. Compared with the traditional space-time method of fundamental solutions, the proposed LSTMFS is more suitable for solving large-scale and long-time diffusion problems. Furthermore, the LSTMFS without temporal-difference is simple, accurate and easy-to-implement due to its semi-analytical and meshless features. Numerical experiments, including diffusion and convection-diffusion problems, confirm the validity and accuracy of the proposed LSTMFS.

AMS subject classifications: 35K15, 35K20, 65M70, 76R50

Key words: Localized space-time method of fundamental solutions, meshless method, time-dependent fundamental solutions, diffusion, convection-diffusion.

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1 Introduction

Diffusion and convection-diffusion models widely exist in many areas of science and engineering applications [1–3]. The numerical simulation for such problems has long been a very attractive area of research [4,5]. For most traditional methods in solving dynamic problems, the difference scheme is often employed to deal with the temporal derivative of the governing equation, which may be time-consuming and may also cause instability if the time step is not carefully chosen. In the spatial discrete methods, the finite difference scheme [6,7], finite element method [8], and boundary element method [9,10] are more commonly used algorithms. These approaches definitely have been both effective and fully play their respective advantages in the numerical simulation. However, they also have some limitations need to be further addressed, such as the troublesome grid generation in grid-dependent methods and the domain/boundary integrals in the boundary element techniques [11,12]. For the simulation of high dimensional dynamic problems in complicated geometries, the existing traditional methods face great challenges, and thus new numerical schemes are encouraged to accurately, efficiently and stably solve the time-dependent problems.

In recent years, many researchers have proposed and developed a variety of meshless/meshfree methods [13–20] in order to reduce or even eliminate the tasks of mesh generation and integral calculation in the conventional numerical techniques. It should be pointed out that most of these methods require the use of differential approximation, dual reciprocity technique or Laplace transform for time derivative in the simulation of transient diffusion and convection-diffusion problems [9,21–23]. This is cumbersome and time-consuming, and may limit the calculation accuracy to some extent. To avoid the above shortcomings, the space-time algorithms [24–26] have already caused researchers' attention.

As an alternative meshless method, the space-time method of fundamental solutions (STMFS) developed by Young et al. [27] is a semi-analytical numerical technique based on the time-dependent fundamental solutions. The method is a mathematically simple and numerically accurate, and does not need the conventionally used Laplace transform or the finite difference scheme to address the temporal derivative in the governing equation. Owing to its merits of simplicity, accuracy and meshless, the STMFS has been successfully applied to the solution of heat conduction [28], wave propagation [29], unsteady Stokes flow [30], fractional anomalous diffusion [31], and so on. The traditional STMFS, however, is a global discretization method with the dense matrix, which restricts its application in the large-scale and long-time dynamic problems. Most recently, the localized space-dependent method of fundamental solutions has been developed to simulate various large-scale steady-state problems [32–34]. To the best of our knowledge, there is no literature reported the localized time-dependent method of fundamental solutions.

Motivated by the above discussion, this study presents a localized space-time method of fundamental solutions (LSTMFS) for transient diffusion and convection-diffusion problems. In the developed LSTMFS, some discrete nodes firstly need to be chosen inside the