Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **12**, No. 4, pp. 972-991

DOI: 10.4208/aamm.OA-2018-0264
August 2020

High Order Hybrid Weighted Compact Nonlinear Schemes for Hyperbolic Conservation Laws

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Received 18 December 2018; Accepted (in revised version) 11 August 2019

Abstract. High order weighted compact nonlinear scheme (WCNS) has become an alternative method of finite difference weighted essentially non-oscillatory (WENO) scheme in many different research areas due to its better spectral properties. However, its heavy computational time even more expensive than the classical WENO scheme is still a bottleneck problem. To relieve it in a sense, a framework of high order hybrid WCNS (HWCNS) combining the weighted nonlinear interpolations proposed in [Deng et al., JCP, 165] or [Zhang et al., JCP, 227] in the non-smooth stencils with corresponding linear compact interpolations in the smooth stencils respectively is designed for solving the hyperbolic conservation laws in this work. A newly developed high order shock detector based on the radial basis function, which can capture the locations of shocks and high gradients accurately and sharply, is used to measure the smoothness of the solution at each grid point. The HWCNS demonstrates higher resolution, lesser dissipation/dispersion errors, lesser computational time in the extensive one- and two-dimensional classical examples by comparing with the WCNS.

AMS subject classifications: 65M10, 78A48

Key words: Hybrid, weighted compact nonlinear scheme, hyperbolic conservation laws, shock detector.

1 Introduction

In the last two decades, the characteristic-wise weighted compact nonlinear scheme (WCNS) [5] was developed for solving the hyperbolic conservation laws in the presence

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of shocks and small scale structures to improve the spectral properties of the classical characteristic-wise finite difference weighted essentially non-oscillatory (WENO) conservative scheme [11]. Then, the higher order WCNS has been developed by Nonomura et al. [20] and Zhang et al. [33] up to the ninth-order, while Zhang et al. [33] proposed interpolating flux instead of the interpolation of conservative variables in the fifth-order and higher order version. Nonomura et al. [21] investigated the effects of the cell-center and cell-node finite difference schemes in the seventh- and ninth-order WCNS. The free-stream and vortex preservation properties on a generalized curvilinear grid are preserved by the WCNS [22]. Deng et al. [7] constructed a class of new high order hybrid cell-edge and cell-node WCNS, the proposed schemes employ more grid points in the compact stencil, but introduce an artificial parameter which can control its dissipation. Nonomura et al. [23] modified the linear finite difference scheme by using the fluxes on the computational nodes together with those on the midpoints, which makes the corresponding scheme more robust, but more dissipative. Yan et al. [31] proposed the new nonlinear weights, which are computed by using not only the ratios, but also the values of smoothness indicators. A new seventh-order compact nonlinear interpolation method was also proposed by Yan et al. [32] on the same stencil as the fifth-order CRWENO scheme used. They used the characteristic projection to interpolations on the substencils and the component-wise interpolation on the whole stencil to avoid solving the expensive block tri-diagnoal matrix. Kamiya et al. proposed the WCNS-CU-Z scheme [13] that adapted the sixth-order central-upwind WENO-CU and fifth-order WENO-Z schemes. Wong et al. [29] developed a WCNS with a sixth-order localized dissipative interpolation, which hybridizes the central interpolation with the more dissipative upwind-biased nonlinear interpolation.

However, the WCNS is highly complex to implement and computationally expensive due to the setup of the Roe-averaged eigensystem etc. It has a procedures of the Riemann solvers for the conservative variable interpolation [5] (WCNS-DZ), or the splitting of the flux into its positive and negative components for the flux interpolation [33] (WCNS-ZSS). A natural remedy is to avoid using the expensive weighted interpolation in the smooth stencils wherever and whenever possible in practice.

In this study, a new high order hybrid WCNS (HWCNS) conjugating the high order linear compact interpolation with the weighted nonlinear interpolation in the smooth and non-smooth stencils respectively is designed for the solution of hyperbolic conservation laws. The newly developed high order and resolution radial basis function shock detector [28], which can detect the locations of shocks and high gradients accurately, is employed to measure the smoothness of solution. For the WCNS-ZSS case, a sixth-order central compact interpolation [17] is used for the flux interpolation in the smooth stencils (HWCNS-ZSS). For the WCNS-DZ case, we interpolate the conservative variables on the cell boundaries through the fifth-order tridiagonal left and right biased compact interpolations [4] in the smooth stencils (HWCNS-DZ). Moreover, the HWCNS-DZ with the HLLC solver (HWCNS-DZ-HLLC) usually behaves better than that with the classical Lax-Friedrichs solver (HWCNS-DZ-LF) in the numerical examples.