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Error Estimation and Stress Recovery by Patch Equilibrium in the Isogeometric Analysis Method

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Abstract. This article is devoted to the development of a new approach for stress recovery in the isogeometric analysis method which makes use of equilibrium of patches. It is shown that the obtained stresses of this approach are more accurate than our previous work based on using the super convergent property of the Gauss quadrature points. To demonstrate the efficiency of this error estimator, a few examples of elasticity problems with available analytical solutions are modeled and solved. The obtained results are compared with the exact solutions as well as our previous method. These results indicate superiority of the current method that can be considered as a simple and efficient approach for stress recovery and error estimation in the isogeometric analysis method.

AMS subject classifications: 74S99

Key words: Isogeometric analysis, error estimation, stress recovery, patch equilibrium.

1 Introduction

Employing the computer aided design (CAD) techniques for engineering analysis was initially attempted by Kaygan and Hollig during the last decade of the past century. In their approach the spline basis functions were employed instead of the finite element shape functions [1–3]. In 2005, the idea was further developed by Hughes and his colleagues who used a further advanced version of splines, i.e. Non Uniform Rational B-Splines (NURBS), and coined the term "isogeometric analysis (IGA)" for their approach [4]. In this method, the spline and NURBS basis functions are used not only for accurate definition of curves, surfaces and volumes, but they are also employed for

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the purposes of interpolation and approximation as well. In the IGA method, despite elimination of the domain modeling errors to a large extent by using NURBS, still there exists some errors in evaluation of displacements. These errors and uncertainties are of larger values for strains and stresses, since these field variables are obtained by taking the derivative of the displacement filed that are approximated with a lower degree of accuracy in comparison to the displacements. Similar to the finite elements, the so named stress recovery methods can be developed to increase the accuracy of the solution.

A few stress recovery techniques have been developed in the past few decades for increasing the accuracy of the solution for stresses in comparison to the initial numerical analysis, as well as obtaining smoother and continuous stress fields. The so called averaging method is amongst the most elementary techniques that was suggested by Hinton and Campbell [5] in 1974. Another method that can be named here is the L2 projection proposed by Oden and Brachelli in 1971 [6]. One of the major advancements in error estimation and stress recovery happened in 1992 when Zienkiewicz and Zhu introduced the superconvergent patch recovery (SPR) method [7]. The main idea in SPR is employing some specific points within finite elements, named as superconvergent points, where the stresses have relatively higher accuracy to create improved stresses. A further progress in the SPR technique was achieved by making use of the equilibrium equations as additional constraints to SPR [8–14]. The inspiration of this approach is due to fact that in the SPR method there is no guarantee that the recovered stresses satisfy the equilibrium equations, especially near the boundaries of the domain of interest.

Stress recovery in the framework of isogeometric analysis method, suggested by Hassani et al. in [15], was based on using the superconvergent property of the Gauss quadrature points of the so called knot elements in analogy to the SPR in finite elements. In this method, an imaginary surface is formed for each component of the improved stress tensor that can also be used for error estimation. Due to the simplicity and efficiency of the method, it was used by many researchers [16–19], but as it was mentioned above, similar to the SPR in FEM, this method suffers from to the fact that the equilibrium equations are not satisfied for the patches of the IGA method.

A novel coupling approach of the isogeometric analysis method and the meshfree method for geometrically nonlinear analysis of thin-walled structures developed by Li et al. in [20]. In this approach the domain is divided into three subdomains: the subdomain described by the IGA method to ensure geometry exactness, the subdomain described by the meshfree method to obtain local refinement, and the coupling subdomain described by both methods. Also, this method was used for static and free-vibration analyses of cracks in thin-walled structures by Li et al. [21].

In this research, by using the forces induced on each patch of IGA, an improved method for computing stresses is introduced that has a better accuracy and can be employed for estimation of the error distribution within the domain. One of the useful features of this method is being independent of the Gauss integration points, that is especially advantageous when a different integration method than the Gauss quadrature is employed.