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Practical Absorbing Boundary Conditions for Wave Propagation on Arbitrary Domain

Fengru Wang[1], Jerry Zhijian Yang[1,2,*] and Cheng Yuan[1]

[1] *School of Mathematics and Statistics, Wuhan University, Wuhan, Hubei 430072, China*
[2] *Computational Science Hubei Key Laboratory, Wuhan University, Wuhan, Hubei 430072, China*

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Abstract. This paper presents an absorbing boundary conditions (ABCs) for wave propagations on arbitrary computational domains. The purpose of ABCs is to eliminate the unwanted spurious reflection at the artificial boundaries and minimize the finite size effect. Traditional methods are usually complicate in theoretical derivation and implementation and work only for very limited types of boundary geometry. In contrast to other existing methods, our emphasis is placed on the ease of implementation. In particular, we propose a method for which the implementation can be done by fitting or learning from the simulation data in a larger domain, and it is insensitive to the geometry and space dimension of the computational domain. Furthermore, a stability criterion is imposed to ensure the stability of the proposed ABC. Numerical results are presented to demonstrate the effectiveness of our method.

AMS subject classifications: 60J05, 62J07, 35L05, 35L20

Key words: Absorbing boundary condition, wave equations, stability conditions.

1 Introduction

Many physical problems involving wave equations require solutions of essentially infinite domain in only a small region, like seismic wave propagation [11], Maxwell equation [23], Poison-Boltzmann equation [18], or inverse problem for the Helmholtz equation [1]. To solve these problems, there are generally two popular tools to overcome the difficulty of the unboundedness of the spacial domain [2,6]. One is the perfectly matched layer (PML) and the other is the artificial boundary method (ABM). The PML, raised by Berenger in 1994 in [4], damps the incident wave in each layer and the performance of

*Corresponding author.
Emails: wangfr@whu.edu.cn (F. R. Wang), zjyang.math@whu.edu.cn (J. Z. J. Yang), yuancheng@whu.edu.cn (C. Yuan)

the absorbing is related to the damping rate and the number of layers [7], and is widely used in engineering like electromagnetic scattering. The main process of ABM is the design of ABCs, which reformulate the unbounded problem into an exact or approximate truncated domain problem. As for building ABCs of wave equations, usually the theoretical analysis plays a major part, like Engquist and Majda approximated the ABCs with a Taylor expansion in [9], and Renaut and Petersen used Chebychev-Padé approximation instead in [25]. Higdon worked directly with the difference scheme in [15, 16], and Shojaei and Mossaiby extended it to finite element method [31]. Comparing to time-domain methods, spectral method also has many advantages like high accuracy and rapid convergence. And ABC concerning frequency domain have also been researched thoroughly [30, 36]. A recent development on this region is the double absorbing boundary [12], which combines the simple formulation of PML and the high-order accuracy of artificial boundary together [24]. All these methods accord with the analytical condition that perfectly absorbs the incident wave at some certain angles of incidence.

However it should be pointed out that those methods have some imperfections in common. In the first place, it is costly in analysis, especially when we face an irregular boundary, like a disk or a polygon region, or when we need a higher-order scheme. It is always not a straightforward replication to popularize the methods, no matter restating the ABC in the new reference axis and approximating it by the finite difference, nor the analysis of its well-posedness. Next, in some cases it seems to be impossible to copy the boundary condition mechanically, like the corners of the rectangle, on account of their bad geometrical symmetry. Hence we have to enlarge the computation region to a more regular one and drop the results in the added region (so-called fictitious domain method). While effective, it may be too computationally expensive. At last, due to that it has been proved that a practical perfectly absorbing boundary condition can never be realized, see, e.g., [13], all these methods aim at approximating and reducing the reflection. Therefore they are effective at some angles of incidence while others not so good. In some situations, we know some information about the incidence beforehand and it limits the absorbing ability not using it. These methods exported from general analysis obviously suffer this problem. In a word, all these methods mentioned above are not problem aimed.

For these reasons, the main purpose of this paper is to give an easy-realizable method of building ABC for wave equations for any region, which we call learning boundary condition (LBC) in this paper, since differing from what we referred above, we obtain our results by machine learning. With the abundant research on the ABC based on functional approximation, the work on this perspective remains open. Our method is easily realizable, can be applied on various shapes of region, and doesn't involve the theoretical analysis of the behavior of the solution to model problems on the boundary, so it can be readily generalized to other PDEs.

The rest of this paper is organized as follows. In Section 2 we educe our method's mathematical formulation. In Section 3, we yield the learning boundary condition for the computational interval $[0,1]$, which equivalents to the exact ABCs given in [9]. In Section