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FEA Versus IGA in a Two-Node Beam Element Based on Unified and Integrated Method

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Abstract. This paper presents a new concept called Unified and Integrated Method for a shear deformable beam element. In this method, Timoshenko beam theory is unified and integrated in such a way that takes into account the effect of transverse shear and maintains the shear locking free condition at the same time to generate proper behavior in the analysis of thin to thick beams. The unified and integrated method is applied to finite element analysis (FEA) and isogeometric analysis (IGA) on two-node beam element. This method will be used to analyze uniformly loaded beams with various boundary conditions. A shear influence factor of ϕ, which is a function of beam thickness ratio (L/h), is expressed explicitly as control of the transverse shear strain effect. The analysis gives interesting results showing that applying the unified and integrated method in FEA and IGA will yield exact values of DOF's and displacement function even when using only a single element. Numerical examples demonstrate the validity and efficiency of the unified and integrated methods.

AMS subject classifications: 65L60, 74-10, 74K10, 74S05, 74S22

Key words: Finite element analysis, isogeometric analysis, unified and integrated method, timoshenko beam theory.

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1 Introduction

Bernoulli-Euler beam theory, also known as classical beam theory, was introduced by Daniel Bernoulli and Leonhard Euler around 1750. It has been widely used to analyze the behavior of the bending element because of its simplicity. The theory assumes that after deformation cross-sections remain plane and orthogonal to the beam axis and that deformation slopes are small. It suggests that shear deformation γ is neglected and rotation θ is equal to the derivative of deflection. Hence, it is more suitable for a slender beam. Vertical deflection v is the only unknown variable is this theory. The curvature in

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the Principal of Virtual Work (PVW) is expressed by the second derivatives of $v(x)$ as the only deformation. This theory requires C^1 continuity to ensure a smooth deflection field.

Timoshenko beam theory [1,2] developed later in the early 20th century offers an improvement. Unlike the classical theory, it takes into account of shear deformation and rotational bending effects so that the previously perpendicular plane sections will not necessarily remain perpendicular to the beam axis after deformation. In this theory, deflection (v) and rotation (θ) are independent of each other. The development of Timoshenko beam element is simpler than of Bernoulli-Euler beam element as it requires C^0 continuity for the deflection and rotation fields. However, the 2-noded Timoshenko beam element suffers from a phenomenon called shear locking when analyzing thin beams. They only provide reasonable solutions in the cases of a thick beam, but give unrealistically stiffer results for thin beams $(L/h>20)$. This phenomenon disqualifies Timoshenko beam elements for the analysis of slender beams.

A popular method to alleviate shear locking in Timoshenko beam elements is by under-integrating the terms in shear stiffness using a quadrature of one order less than needed for exact integration. This method reduces the effect of the transverse shear stiffness and yields constant transverse shear strains along the beam. The terms in bending stiffness are still integrated exactly. This method is known as Selective Reduced Integration (SRI).

There are many methods to eliminate shear locking, one of which is the Assumed Natural Strain (ANS). A number of Timoshenko beam elements have adopted the ANS to deal with the shear locking problem. By applying the ANS, the transverse shear strain in a beam element with two nodes and two degrees of freedom per node becomes constant along with the element [3]. The two-node element with linear interpolation demonstrates satisfactory outcomes over a wide variety of length to thickness ratio. Yet, in the matter of convergence speed, it cannot be compared with the Bernoulli-Euler element, which neglects the shear deformation.

While developing Discrete Shear Gap (DSG) method to overcome shear locking, Bletzinger et al. [4] also applied the ANS concept. DSG beam element satisfies the kinematic equation for the shear strains at discrete nodes and significantly reduces the shear strains. The key of the DSG method is calculating the discrete shear gap at nodes and interpolating them across the element domain. Just like reduced integration, the application of the DSG concept in the beam element with 2 nodes gives a constant shear along the beam.

It is well known that it is possible to derive a 2 nodes beam element that gives exact results (at least at nodes) based on mixed formulation [5]. Exact here means that the results are valid for thick to thin beams, without the occurrence of shear locking.

Another beam element that adopts the ANS concept is Discrete Shear Beam (DSB). DSB element [6] uses cubic interpolations to calculate total vertical displacement (v) and quadratic interpolations to calculate rotation (θ). In this element, the transverse shear strain is defined as constant along the beam by using the discrete shear method. DSB element has been the basis of the development of triangular DKMT and quadrilateral DKMQ plate and shell elements [7–17]. However, besides the good performance over