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Legendre Neural Network for Solving Linear Variable Coefficients Delay Differential-Algebraic Equations with Weak Discontinuities

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Abstract. In this paper, we propose a novel Legendre neural network combined with the extreme learning machine algorithm to solve variable coefficients linear delay differential-algebraic equations with weak discontinuities. First, the solution interval is divided into multiple subintervals by weak discontinuity points. Then, Legendre neural network is used to eliminate the hidden layer by expanding the input pattern using Legendre polynomials on each subinterval. Finally, the parameters of the neural network are obtained by training with the extreme learning machine. The numerical examples show that the proposed method can effectively deal with the difficulty of numerical simulation caused by the discontinuities.

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Key words: Convergence, delay differential-algebraic equations, Legendre activation function, neural network.

1 Introduction

Delay differential-algebraic equations (DDAEs) arise in many areas of mathematical model, such as circuit design, mechanical system, power system, control theory [1], multi-body control system, bioeconomic system [2], fluid mechanics [3], chemical engineering. The reference [4] has indicated that differential-algebraic equations (DAEs) are neither differential equations nor algebraic equations. Differential equations only involve differentiation, while DAEs contain differentiation and integration, which changes the behavior of the

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solution [5], so the numerical methods for solving differential equations can not be directly applied to solve DAEs. Furthermore, DDAEs are not only restricted by algebraic conditions but also affected by time delays. Bellen and Zennaro [6] have verified that delays can cause discontinuities and affect stability of the solution. Thus, algebraic conditions and delays cause some difficulties in numerical simulation of DDAEs. In recent years, many documents on its characteristics and theories of DDAEs have been discussed in [8–13], stability results of numerical methods for DDAEs have been presented in [14–21], and convergence analysis of numerical methods for DDAEs have been described in [22–26]. Furthermore, Ascher and Petzold [27] developed a numerical approach of high index DDAEs. The above works are under the basis of the solutions being smooth. Few studies have been carried out on the effects of DDAEs with weak discontinuities [28].

Neural networks have been widely used in the solution of mathematical physics problems. Now popular neural networks include feedforward neural network, radial basis function neural network, cosine basis function neural network, diagonal recurrent neural network, cellular neural network, finite-element neural network, etc. The neural network can obtain the weights and structure of the network by training and learning, showing strong self-learning and adaptive ability. That is, the network structure, node weights and step sizes can be automatically adjusted according to environmental requirements. An advantage of neural networks to solve differential equations is that the solution of the differential equations can be expressed as a differential function. Earlier in 1992, Shelton et al. [29] applied neural network to deal with the coupled nonlinear ordinary differential equations, and it was also used to solve (non) linear ordinary differential equations [30–36], partial differential equations [37–44], delay differential equations [45,46], stiff differential equations [47, 48], stochastic differential equations [49] and fractional differential equations [50–52]. Kozlov and Tiumentsev [53] introduced a neural network based on semi-empirical models for solving DAEs of index 2. Yang et al. [54] solved DAEs based on the artificial neural network. Few scholars manage to solve DDAEs with weak discontinuities at present.

Compared with the traditional methods, the advantages of the Legendre neural network (LNN) to solve DDAEs are as follows [55, 56]. (1) LNN can overcome the iterative process commonly used in traditional numerical methods. (2) The computational complexity of LNN does not increase quickly with the increase of sample points. (3) LNN has fault tolerance and tolerance capabilities. The contribution of each neuron and each connection to the overall network function is small, so the failure of a few neurons and connections has little effect on the network function. Based on the above advantages, in this paper, LNN [58] combined with the extreme learning machine (ELM) algorithm [59,60] is applied to solve the variable coefficients linear DDAEs with weak discontinuities. Firstly, the interval is divided equally into multiple subintervals according to the weak discontinuity points. Secondly, we eliminate the hidden layer by using the Legendre basis function that is used to extend the input pattern. Thirdly, the weights of LNN are obtained by ELM training neural network on each subinterval. Finally, we get the approximate solutions of the DDAEs on the whole interval.