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# The Fictitious Domain Method with Sharp Interface for Elasticity Systems with General Jump Embedded Boundary Conditions

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**Abstract.** In framework of the fictitious domain methods with immersed interfaces for the elasticity problem, the present contribution is to study and numerically validate the jump-integrated boundary conditions method with sharp interface for the vector elasticity system discretized by a proposed finite volume method. The main idea of the fictitious domain approach consists in embedding the original domain of study into a geometrically larger and simpler one called the fictitious domain. Here, we present a cell-centered finite volume method to discretize the fictitious domain problem. The proposed method is numerically validated for different test cases. This work can be considered as a first step before more challenging problems such as fluid-structure interactions or moving interface problems.

**AMS subject classifications**: 65M55, 65N08, 65N85

**Key words**: Fictitious domain method, sharp interface, elasticity system, jump embedded boundary conditions, finite volume method.

## 1 Introduction

The conventional grid method approach to simulating transmission problems in heterogeneous elastic body with complex boundaries is to discretize the governing equations on a curvilinear mesh that conforms to the boundaries. The main advantages of this approach are that imposition of boundary conditions is greatly simplified and the scheme

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can be easily designed so as to maintain sufficient accuracy and conservation property. However, the geometrical complexity of immersed boundaries can adversely impact the stability, the convergence and the operation count of the solver. An alternative method approach which is gaining popularity in recent years is the fictitious domain methods approach where the governing equations are discretized on a uniform structure Cartesian mesh which does not conform to the immersed boundaries. This greatly simplifies mesh generation and retains the relative simplicity of the governing equations in Cartesian coordinates, (see [1–8, 10, 11]). In addition, since the Cartesian mesh scheme does not depend on the location of the immersed boundary, there is no need for remesching strategies and, thus, has a significant advantage in simulating problems with moving boundaries and complicated shapes. However, the difficulty in using the fictitious domain method in conjunction with a uniform structured Cartesian mesh scheme is in the position of the original boundary conditions at the immersed boundaries. Thus, to avoid this complication, an approximate interface $\Sigma_h$ of the original $\Sigma$ is constructed by a series of cell-sides that are cut by the boundary on which we apply an algebraic transmission boundary conditions proposed in our fictitious domain model-problem.

The numerical validation of a new fictitious domain method associated with general Jump Embedded Boundary Conditions (J.E.B.C) is proposed in the present paper. To solve the linear elasticity system governed by the given problem $(\widetilde{P})$ in the original domain $\widetilde{\Omega}$, a fictitious domain technique is used. The key issue here is to construct an extended imperfect transmission problem $(P)$ of $(\widetilde{P})$ defined in the extended domain $\Omega$ of the original physical domain $\widetilde{\Omega}$ in which its geometric shape is simpler than that of $\widetilde{\Omega}$ such that

$$\Omega=\widetilde{\Omega}\cup\Sigma\cup\Omega_e,$$

where $\Omega_e$ is the external domain and $\Sigma$ the common interface between $\widetilde{\Omega}$ and $\Omega_e$, see Fig. 1. With an appropriate choice of the data and the transmission conditions in the auxiliary domain $\Omega_e$ and on the interface $\Sigma$ respectively, the transmission problem $(P)$ will be well-posed and the two problems are equivalents in the following sense : If $\mathbf{u}$ is a solution of the fictitious problem $(P)$ defined in the fictitious domain $\Omega$, then the restriction of the fictitious solution

$$\tilde{\mathbf{u}}=\mathbf{u}|_{\widetilde{\Omega}}$$

or, at least, that $\widetilde{\mathbf{u}}_\eta=\mathbf{u}|_{\widetilde{\Omega}}$ is a solution of the physical problem (original) $(\widetilde{P})$.

Our objective is to use a simple structured mesh in $\Omega$, e.g., a uniform Cartesian grid, independent of the shape of the immersed interface $\Sigma$, instead of using an adaptive mesh to which it becomes difficult to find fast and efficient solvers. The method is applied to several test cases for which an analytical and finite volume solution exists. Comparisons of the numerical and analytical results show a very good performance of the method. Our quest is to solve, with a fictitious domain method in $\Omega$, the following problem originally defined in $\widetilde{\Omega}\subset\Omega$ with a general boundary condition on $\Sigma$ : Find the displacement vector