Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **13**, No. 1, pp. 163-175

DOI: 10.4208/aamm.OA-2019-0188
February 2021

Numerical Solution of Steady-State Free Boundary Problems using the Singular Boundary Method

Fen Chen[1,2], Bin Zheng[3,*], Ji Lin[3] and Wen Chen[3]

[1] *College of Civil Engineering, Hefei University of Technology, Hefei, Anhui 230009, China*
[2] *College of Civil Engineering, Taizhou University, Taizhou, Zhejiang 318000, China*
[3] *College of Mechanics and Materials, Hohai University, Nanjing, Jiangsu 211100, China*

Received 24 June 2019; Accepted (in revised version) 17 September 2019

Abstract. In this paper, the recently-developed singular boundary method is applied to address free boundary problems. This mesh-less numerical method is based on the use of the origin intensity factors with fundamental solutions. Three numerical examples and their results are compared with the results obtained using traditional methods. The comparisons indicate that the proposed scheme yields good results in determining the position of the free boundary.

AMS subject classifications: 00A69

Key words: Seepage flow, singular boundary method, mesh-less, origin intensity factors.

1 Introduction

Water seepage is the leading cause of a considerable number of geological disasters. The determination of the free boundary for the seepage plays a major part in this problem. This problem has been extensively studied by variable grid methods, fixed grid methods, and mesh-less methods in recent decades.

Liao proposed the residual pressure feedback method [1] for the simulation of free surface flow. Neumann put forward the Galerkin method [2] which was continuously optimized and developed as an element-free method [3]. Subsequently, a large variety of grid methods attempted to detect the location of free boundary, such as the residual flow method [4], the osmosis matrix adjustment [5], the initial flow method [6], the variational inequality [7], and the finite element method (FEM) [8]. The disadvantages of these methods are that they are time-consuming and inflexible.

*Corresponding author.
Email: zhengbinyilan@163.com (B. Zheng)

 ©2021 Global Science Press

Based on a set of scattered nodes, mesh-less methods overcome certain limitations of grid methods. The Kansa method [9], the boundary knot method (BKM) [10], the backward substitution method [11, 12], and the method of fundamental solutions (MFS) [13–15] are such methods. Nevertheless, the MFS shows superiority of stability over the LRBF.

Chen and his collaborators devised a new boundary-type mesh-less method, namely the singular boundary method (SBM), which is a coupling between the MFS and the indirect boundary element method (BEM) [16]. The SBM employs the fundamental solutions as the basis functions, and introduces the concept of origin intensity factors [17,18] to take place of the singularities encountered in the fundamental solutions at origin. The method inherits the dimensional superiority of the BEM and does not involve costly integration. The method eliminates the complicated fictitious boundary issue associated with the traditional MFS. Based on the SBM, highly accurate results have been obtained in various wave cases [19–21], large-scale problems [22], transient matters [23,24], and heat conduction problems in layered materials [25, 26]. In this paper, the SBM is extended to solve steady-state free boundary problems.

The rest of the paper is organized as follows. In Section 2, we give a brief review on the basic idea of the SBM for solving steady-state free boundary problems. In Section 3, in order to demonstrate the effectiveness of the SBM, three numerical examples are presented. A summary and conclusions are provided in the last section.

2 The SBM for the free boundary problems

We consider the following problem [27]

$$\nabla^2\phi(\mathbf{x})=0, \quad \mathbf{x}\in\Omega, \tag{2.1}$$

subject to the boundary conditions:

$$B\phi(\mathbf{x})=f, \qquad \mathbf{x}\in\partial\Omega_{\text{FIXED}}, \tag{2.2a}$$

$$\begin{cases} B_1\phi(\mathbf{x})=f_1, \\ B_2\phi(\mathbf{x})=f_2, \end{cases} \qquad \mathbf{x}\in\partial\Omega_{\text{FREE}}, \tag{2.2b}$$

where $\partial\Omega=\partial\Omega_{\text{FIXED}}\cup\partial\Omega_{\text{FREE}}$, $\partial\Omega_{\text{FIXED}}$ is the fixed boundary and $\partial\Omega_{\text{FREE}}$ is the free boundary. B denotes the boundary operator on the fixed boundary. The two different kinds of boundary operators on the free boundary are denoted by B_1 and B_2. f, f_1 and f_2 represent known functions. The least squares fit to the data generated is treated as follows :

$$F=\left\{\sum_{m=1}^{M_{\text{FIXED}}}|B\phi(\mathbf{x}_m)-f|^2+\sum_{m=1}^{M_{\text{FREE}}}\Big[|B_1\phi(\mathbf{x}_m)-f_1|^2+|B_2\phi(\mathbf{x}_m)-f_2|^2\Big]\right\}^{\frac{1}{2}}. \tag{2.3}$$

$\phi(\mathbf{x})$ is an approximate solution with unknown coefficients. M_{FIXED} and M_{FREE} denote the number of fixed boundary and free boundary nodes, respectively. In this paper, the