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An Implicit-Explicit Scheme for the Radiation Hydrodynamics

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Abstract. In this paper, we study an implicit-explicit scheme for the radiation hydrodynamics in the equilibrium diffusion limit and in the grey nonequilibrium diffusion limit. We extend a popular Godunov-type method, the MUSCL-Hancock scheme, to the convective part of the radiation hydrodynamics, while a cell centered finite volume scheme is used for the radiative heat transfer. Moreover, the implicit-explicit scheme is easier to implement. Numerical simulations show the character of the radiative shock wave and the accuracy of the scheme.

AMS subject classifications: 65M06

Key words: Radiation hydrodynamics, MUSCL Hancock scheme, cell centered finite volume scheme, implicit-explicit scheme, radiative shock.

1 Introduction

The radiation hydrodynamics (RH) has been playing an indispensable role in astrophysics, inertial confinement fusion and some other high-temperature systems. It describes the propagation of thermal radiation through a fluid and the influences of the radiation on fluid motion. The exchanges of momentum and energy between the matter and radiation field result from the processes of the absorption, scattering, and emission of photon [11, 12]. In the radiation hydrodynamics, the multiple time scales and the stiff source bring difficulties for developing a numerical scheme. In order to overcome the difficulties caused by the stiff source, we may split the radiation hydrodynamical equations into a hyperbolic system of conservation laws and a nonlinear radiative heat transfer. Material quantities are characterized by waves propagating at the speed of sound a_∞, while the radiation quantities are characterized by waves propagating at the speed of light c. Thus, for the radiation system, we build the implicit-explicit finite volume

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scheme. The fluid hydrodynamics is treated explicitly, and the radiation heat transfer is treated implicitly.

When the radiation coefficient becomes zero, the fluid hydrodynamics would reduce to the Euler equations in gas dynamics. For the Euler equations, a large number of high order accurate numerical methods have been got great development [7, 15]. Godunov proposes the method of using piecewise constant data as initial data for the numerical calculation of Euler equations of the fluid dynamics in [5]. Van Leer develops the MUSCL scheme [17], by piecewise linear data. Then a lot of high order accurate schemes have appeared, such as the GRP.

Due to the radiation effect, developing a numerical scheme for the radiation hydrodynamical equations is more difficult than for the Euler equations. For the RH in the zero diffusion limit, Tang and Wu apply the kinetic flux-vector splitting method to the RH model [14]. Kuang and Tang develop the direct Eulerian GRP scheme for this RH model in [6]. For the RH in the equilibrium diffusion limit, Dai and Woodward propose a Godunov-type scheme using linear and nonlinear Riemann solvers in [4]. Qamar and Ashraf apply a central schemes to solve the equilibrium model [13]. Lowrie and Rauenzahn propose a semi-analytic solution for the shock wave in the equilibrium diffusion limit [8]. For the nonequilibrium diffusion limit, Chauveheid et al. apply the characteristic flux finite volume scheme to the hydrodynamics part in [3]. Lowrie and Edwards advance a semi-analytic solution of the radiative shock waves for the grey nonequilibrium diffusion RH model [9]. In this paper, we extend a popular Godunov-type method, the MUSCL-Hancock method, to the hydrodynamics part of the RH models in the equilibrium diffusion limit and in the grey nonequilibrium diffusion limit, and the method is easy to implement. Taking into account the problem of the small time scales for the radiative heat transfer, we adopt an implicit centered finite volume scheme. Numerically, the hydrodynamics part and radiative heat transfer are solved continuously, which yields the solutions of RH models.

The organization of this paper is as follows. Section 2 introduces the radiation hydrodynamical equations in the equilibrium diffusion limit and in the grey nonequilibrium diffusion limit. Furthermore, we compare the differences in the structure of the radiative shock between the equilibrium and non-equilibrium models in this section. In Section 3, an implicit-explicit scheme is applied to the radiation hydrodynamical equations. Numerical experiments show the character of the radiative shock wave and the accuracy of the scheme in Section 4. Section 5 is the conclusion for this paper.

2 The radiation hydrodynamics model

If assumed that a single material is radiative opaque. The nonrelativistic equations of the radiation hydrodynamical are written in the grey nonequilibrium diffusion form [9],

$$\partial_t \rho + \partial_x(\rho u) = 0, \tag{2.1a}$$