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An Improved Single-Relaxation-Time Multiphase Lattice Boltzmann Model for Multiphase Flows with Large Density Ratios and High Reynolds Numbers

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Abstract. In this study, an improved single-relaxation-time multiphase lattice Boltzmann method (SRT-MLBM) is developed for simulating multiphase flows with both large density ratios and high Reynolds numbers. This model employs two distribution functions in lattice Boltzmann equation (LBE), with one tracking the interface between different fluids and the other calculating hydrodynamic properties. In the interface distribution function, a time derivative term is introduced to recover the Cahn-Hilliard equation. For flow field, a modified equilibrium particle distribution function is present to evolve the velocity and pressure field. The present method keeps simplicity of the conventional SRT-MLBM but enjoys good stability property in simulating multiphase. Apart from several benchmarks, the present model is validated by simulating various challenging multiphase flows, including two droplets impact on liquid film, droplet oblique splashing on a thin film and a drop impact on a moving liquid film. Numerical results show the reliability of present model for effectively simulating complex multiphase flows at density ratios of 1000 and high Reynolds numbers (up to 7000).

AMS subject classifications: 76T10, 76D05, 76M28, 76P99

Key words: Lattice Boltzmann model, multiphase flow, large density ratio, high Reynolds numbers.

1 Introduction

In the past decades, the lattice Boltzmann method (LBM), is widely used to study multiphase flows due to its mesoscopic features [1–12]. From the physical model it based

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on, the popular LBM for the multiphase flows (indicated as MLBM hereafter) include the color-gradient model [1], the pseudo-potential model [2, 3], the free energy model [4, 5] and the interface tracking model [6, 7]. Although the MLBMs have already achieved tremendous successes in modeling the multiphase flows, there are still plenty of potential challenges in simulating the problems with complex interface variation, large density and viscosity ratios, [8, 13–17], especially for the SRT-MLBM.

For most environmental flows, the density ratios of different phases can be of the order of 1000 : 1. Thus, a reliable MLBM should be able to solve the challenge problems with such conditions. Within this context, Inamuro et al. [13] first proposed a SRT-MLBM based on the free-energy model and achieved satisfactory results with large density ratio. Their model employed two distributions to evolve the density and the predicted velocity. However, the pressure had to be corrected by Poisson equation in every step, which spoiled the simplicity of the LBM. To overcome this drawback, Inamuro et al. [14] further proposed an improvement model without considering the Poisson equation in recent years. However, it just shown the ability for the density ratios of 855 : 1. Lee and Lin [15, 16] introduced a stable SRT-MLBM based on He's double distribution functions model [6, 7]. Their methods used three steps to solve LBE, and a combined finite difference scheme was particularly designed with the idea of fractional-step to ensure the numerical stability. Unfortunately, the mixed scheme was quite complex in implementation, and could also cause the violation of mass conservation [18,19]. Moreover, the Cahn-Hilliard equation in their model that developed for interface tracking wasn't recovered from the LBE exactly [17, 20]. To solve this problem, a special finite difference scheme was designed in the interface distribution function [20]. Based on this work, Zheng et al. [17] further presented a MLBM for the large-density-ratio multiphase flows. However, a significant difference was noted from the results of Fakhari and Rahimian [21], and they found that the method of Zheng et al. [17] was limited to density-matched binary fluids. To eliminate such defects, a free-energy-based MLBM was introduced [22]. In this model, the local density and momentum of flow were considered by introducing an incompressible transformation [7, 15] in the particle distribution function. As a result, the model of Shao et al. [22] correctly simulated the effect of density contrast in the momentum equation, but it unfortunately lost the capacity of simulating multiphase flow with large density ratio. To improve the physical diffusion and numerical dissipation of Shao's model [22], Niu et al. [23] introduced a mass correction term in the interfacial LBE. Despite the great performance in mass conservation, their model also fails to simulate large-density-ratio multiphase flow problem. Due to its high numerical stability, the multi-relaxation-time based LBM were developed to model multiphase flows and becomes a major approach for solving large-density-ratio problem [24, 25]. Moreover, a series of novel treatment, such as lattice Boltzmann flux solve [26] and simple lattice Boltzmann method [27], were used to simulate large density ratio problem with high accuracy [28, 29].

As a phase-field equation choice, the second-order Allen Cahn equation is also used to track the motion and topological change of the interface and becomes a hot spot of