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Upwind Strategy for Localized Method of Approximate Particular Solutions with Applications to Convection Dominated Diffusion Problems

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Abstract. A novel upwind technique for the localized method of approximate particular solutions (LMAPS) is proposed to solve the convection-diffusion equations. An upwind approximation to the convective terms is implemented by choosing upwind interpolation stencils while the central interpolation stencils are used for the diffusive terms. The proposed upwind LMAPS scheme is also been compared with conventional LMAPS without upwind technique, to demonstrate its superiority in generating high accurate solutions than the latter. Numerical results show that the proposed upwind LMAPS has high accuracy and efficiency for a variety of convection-diffusion equations.

AMS subject classifications: 65M10, 65M70, 35L03

Key words: LMAPS, convection dominated problems, upwind scheme, radial basis functions.

1 Introduction

The meshless methods based on radial basis functions (RBFs) such as radial basis function collocation method (RBFCM) [1], differential quadrature (DQ) method [2, 3] and the method of approximate particular solutions (MAPS) [4–6] are successfully applied to solve various partial differential equations (PDEs) including convection-diffusion problems [7], Burgers' equations [8], Navier-Stokes equations [9] and turbulent flow [10], etc. It is worth noting that the meshless methods are used in a global manner in classic collocation technique, so the resultant coefficient matrices may be subjected to singular, dense and even ill-conditioned problems which restricts our abilities to solve large-scale problems. Consequently a new class of effective constructions were proposed to overcome these shortcomings. Among these new techniques, local scheme based on local support interpolations appears to be more efficient in handling a large number of collocation points and less sensitive to changes of shape parameter [8, 9].

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The convection-diffusion equations is a kind of mathematical model widely used in fluid dynamics and thermodynamics. Due to its physical significance, many researchers pay close attention to it [11–14]. However, the most common RBFs interpolation formulation to solve convection dominated problems uses the central type of stencils in the local support domain, without considering the direction and the magnitude of the convective flow. The convection term causes spurious oscillations around the discontinuous solutions that span several grids in finite time steps when using classical numerical techniques based on central interpolation stencils. The development of efficient and accurate RBFs meshless methods for the simulation of convection dominated flows is therefore a challenged research topic. The upwind schemes are often regarded as one of the most efficient methods. The purpose of this paper is to formulate an upwind LMAPS scheme and to extend it to solve convection-diffusion problems.

The paper is organized as follows. The governing equations of the problems are given in Section 2. In Section 3, the introduction to the LMAPS as well as the upwind formulation of the LMAPS is presented. The numerical results are compared with those obtained by other methods or analytical solutions in Section 4. Finally, conclusions are drawn in Section 5.

2 Governing equations

Without loss of generality, we consider the following time-dependent convection-diffusion equation

$$\frac{\partial \mathbf{u}}{\partial t} - \varepsilon \Delta \mathbf{u}(\mathbf{x},t) + \beta \cdot \nabla \mathbf{u}(\mathbf{x},t) + \gamma \mathbf{u}(\mathbf{x},t) = f(\mathbf{x},t), \quad (\mathbf{x},t) \in \Omega \times (0,T], \tag{2.1a}$$

with initial condition

$$\mathbf{u}(\mathbf{x},0) = \mathbf{u}_0(\mathbf{x}), \quad \mathbf{x} \in \Omega, \tag{2.1b}$$

and boundary condition

$$\mathbf{u}(\mathbf{x},t) = g(\mathbf{x},t), \quad (\mathbf{x},t) \in \partial\Omega \times (0,T], \tag{2.1c}$$

where $\varepsilon > 0$ is the diffusion coefficient that corresponds to the Peclet number, β is a known convection velocity, γ is reaction coefficient and f is a known source term. $\mathbf{x} \epsilon \Omega \subset \Re^d$, $d = \{1,2,3\}$, is a bounded domain with the boundary $\partial\Omega$. The convective term plays a dominant role in convection-diffusion equation which behaves as hyperbolic type of partial differential equation as the diffusion coefficient ε tends to zero. As a result, traditional meshless methods can not produce monotonic solutions and suffer from the spurious numerical oscillations around the discontinuous solutions.