Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **13**, No. 2, pp. 467-480

DOI: 10.4208/aamm.OA-2018-0149
April 2021

# Effect of Leading-Edge Curvature on Receptivity of Stationary Cross-Flow Modes in Swept-Plate Boundary Layers

Luyu Shen and Changgen Lu*

*School of Marine Sciences, Nanjing University of Information Science & Technology,*
*Nanjing, Jiangsu 210044, China*

Received 25 June 2018; Accepted (in revised version) 8 June 2020

**Abstract.** The prediction and control of the laminar-turbulent transition in three-dimensional boundary layers are crucial for the designs of vehicles, aircrafts, etc. Receptivity, the initial stage of transition, is the key to implement the prediction and control of the transition process. Former experimental results showed that, under a relatively low-level turbulence, the three-dimensional boundary-layer transition is mainly induced by the stationary cross-flow modes rather than the travelling ones. Near the leading edge, stationary cross-flow modes can be excited by three-dimensional localized roughness. And the receptive process is affected by both the size of roughness and the shape of leading edge which distorts the mean flow over the plate. Therefore, we perform direct numerical simulation to investigate the excitation of stationary cross-flow modes by three-dimensional localized wall roughness in swept-plate boundary layers with various elliptic leading edges. The effect of the leading-edge curvature on the induced stationary cross-flow modes is revealed. And the relations of the leading-edge curvatures with the amplitudes and dispersion relations of the stationary cross-flow modes are determined. Furthermore, the correlations between the receptivity coefficients and the geometries, locations and numbers of the roughness respectively are analyzed in different leading-edge curvatures. This research aims to complement the study of receptivity in the cross-flow boundary layer.

**AMS subject classifications**: 76D05, 76F65

**Key words**: Receptivity, leading-edge curvature, stationary cross-flow mode.

## 1 Introduction

The prediction and control of the laminar-turbulent transition have long been a hot issue

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*Corresponding author.

*Emails:* shenluyu@nuist.edu.cn (L. Y. Shen), cglu@nuist.edu.cn (C. G. Lu)

pursued by a number of scholars. The process of the laminar-turbulent transition is extremely complex. And receptivity is the first stage in the transition and hence the key to the prediction and control of the transition. The early researches involved with receptivity focused on two-dimensional boundary layers, while most of the receptivity in engineering occur in three-dimensional boundary layers, such as a swept-wing surface, a cone with an angle of attack and the turbine blades etc. It has been experimentally confirmed that under a relatively high-level turbulence, the three-dimensional boundary-layer transition is dominated by the travelling cross-flow modes [1, 2]. However, the actual free flight is in an environment of low-level turbulence that three-dimensional boundary-layer transition is dominated by the stationary cross-flow modes [1]. Therefore, the research on the receptivity of stationary cross-flow modes in the three-dimensional boundary layer is necessary for the design of the aircrafts.

Bippes and Nitschke-Kowsky [2] first experimentally proved that the stationary cross-flow modes in three-dimensional boundary layers are induced by the wall roughness rather than the free-stream disturbances. And Radeztsky et al. [3] also found in experiments that the three-dimensional boundary-layer transition dominated by stationary cross-flow modes is triggered by the three-dimensional localized wall roughness. Subsequently, a series of investigations were carried out by Radeztsky et al. [3, 4], Deyhle and Bippes [5], and Reibert et al. [6, 7] to ascertain the relation between the geometries, sizes and locations of the three-dimensional roughness and the three-dimensional boundary-layer receptivity separately. And the nonparallel effect on the three-dimensional boundary-layer receptivity was computed by Betrolotti [8] and Collis and Lele [9]. By using direct numerical simulation (DNS) and parabolized stability equations (PSE), Tempelmann et al. [10] simulated the three-dimensional boundary-layer receptivity under three-dimensional localized wall roughness, and the obtained results agree with the experimental data from Reibert et al. [6]. Recently, Kurz and Kloker [11] studied the relationship between the three-dimensional boundary-layer receptivity and roughness height.

Receptivity of three-dimensional boundary layers has been theoretically studied as well. The mathematical formulations of finite-Reynolds-number were adopted first. Crouch [12] and Choudhari [13] both considered excitation of stationary vortices by spanwise-periodic surface roughness in the Falkner-Skan-Cooke flow. The boundary-layer response has continuous streamwise spectra, from which an unstable vortex mode would emerge. Furthermore, Ng and Crouch [14] considered receptivity of the boundary layer over a swept wing. A good agreement was found between their theoretical prediction and the measurement for roughness height up to twice of the local boundary-layer displacement thickness.

Complementary to the finite-Reynolds-number studies, large-Reynolds-number asymptotic analysis provides deeper insights into the mechanisms of cross-flow instability. Choudhari [15] revealed that the linear development of stationary vortices fall into