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A Radial Basis Function Meshless Numerical Method for Solving Interface Problems in Irregular Domains

Xin Lu[1], Ping Zhang[1], Liwei Shi[2,*], Songming Hou[3] and Ying Kuang[1]

[1] *Department of Mathematics, College of Science, China University of Petroleum (Beijing), Beijing 102249, China*
[2] *Department of Science and Technology, China University of Political Science and Law, Beijing 102249, China*
[3] *Department of Mathematics and Statistics and Center of Applied Physics, Louisiana Tech University, Ruston, LA 71272, USA*

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Abstract. In this paper, we use the radial basis function meshless method to solve the irregular region interface problem. The key idea is to construct radial basis functions corresponding to different regions divided by the interfaces. This method avoids the difficulty of mesh generation, and is efficient in the numerical simulation of partial differential equations in irregular domain with variable matrix coefficients. The numerical error is effectively reduced by using the direct method to handle the interface jump conditions. Numerical simulation results show that the radial basis function meshless numerical method can effectively deal with various kinds of interface problems with irregular domains and sharp-edged interfaces, including Poisson equations, heat conduction equations and wave equations.

AMS subject classifications: 65N30

Key words: Interface problem, irregular domain, matrix coefficient, partial differential equation, meshless method.

1 Introduction

The numerical solution of interface problems has attracted much attention for their wide applications. The band structure computation of composite materials in material science [1–4], the miscible driving problem in fluid dynamics [5, 6], and the solvation of biomolecules and solvent molecules in molecular biology [7, 8] are some examples of interface problems. Naturally, great quantities of numerical schemes are proposed for

*Corresponding author.
Emails: xinlu@cup.edu.cn (X. Lu), zhangpingby@126.com (P. Zhang), shiliweita@126.com (L. W. Shi), shou@latech.edu (S. M. Hou), kuangying_kay@163.com (Y. Kuang)

interface problems, which are based on one or more of Finite Element Methods (FEMs), Finite Difference Methods (FDMs), Finite Volume Methods (FVMs) and Meshless methods.

FEMs are efficient approaches for the interface problems and attract widespread attention. The extended finite element method (XFEM) [9] enriches the standard FEM by adding special basis functions to overcome the discontinuity. The immersed finite element method (IFEM) [10] revises basis functions to satisfy the homogeneous jump conditions. In [11], the numerical solution is continuous inside each element but not on the boundary of each element. Based on IFEM, some improved versions, such as the Petrov-Galerkin IFEM [12], symmetric and consistent IFEM [13] and partially penalized IFEM [14], are constantly emerging. In [15, 16] A. Hansbo and P. Hansbo constructed an approximate solution on each fictitious domain and use Nitsche's technique to patch them together. In [17], the authors studied a class of discontinuous Galerkin (DG) methods for elliptic interface problems. Later, a high-order HDG method was proposed by Huynh in [18]. Recently, S. Hou and L. Wang proposed a non-traditional finite element formulation achieving second-order accurate for sharp-edged interfaces and extended it to general elliptic equations in [12] and [19]. It can also be used in combination with other methods to ensure accuracy [20] and solve variable coefficient wave equations with jump conditions [21].

Associated with the framework of FDM, using difference scheme near the interface to unite jump conditions and achieve high-order local truncation error, several numerical methods have been proposed for interface problems. Peskin originally proposed the immersed boundary method (IBM) to simulate blood flow in [22]. The key to IBM is to use the Dirac δ-function to model the discontinuity and discretize it. For the reason that IBM can only achieve the first-order accuracy, Leveque and Li characterized the discontinuity as jump conditions and proposed the immersed interface method (IIM) [23], which constructs special finite difference schemes to incorporate the jump conditions near the interface. In 1994, a level set method [24] combined with the immersed boundary method was proposed by Osher in order to compute the incompressible two-phase flows. This level set method is easy to implement, although it is of low convergent rate. The ghost fluid method (GFM) is another promising method for interface problems as a pioneering work of Liu, Fedkiw, Myungjoo and Kang [25]. Although GFM only needs to modify the right-hand side, it is of first-order accuracy. To obtain second-order accuracy, a local Voronoi grid is used near the irregular domain's boundary in a recent work [26]. In [27], Epshteyn designed a high-order accuracy Difference Potentials Method (DPM) for some types of interface problems. Recently, some novel methods have been posed, such as the matched interface and boundary Method (MIBM) [28] and the virtual node method(VNM) [29].

FVM is very useful in conservation law problems, like computational fluid dynamics [30] and many physical models. In [31], the FVM and IFEM are combined to get an FVE method, which can solve elliptic interface problems on triangular meshes with homogeneous jump conditions. A bilinear IFVE method on rectangular meshes was proposed in [32]. The finite volume characteristics flux with improved natural interface posi-