

A Multi-Dimensional Shock-Capturing Limiter for High-Order Least Square-Based Finite Difference-Finite Volume Method on Unstructured Grids

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Abstract. In this paper, a simple and robust multi-dimensional shock-capturing limiter is presented and applied to the high-order least square-based finite difference-finite volume (LSFD-FV) method for simulation of compressible inviscid flows on unstructured grids. The novel limiter considers all the spatial information of the neighbouring supporting cells of the reference cell instead of only the immediate neighbours. As a result, the local upper and lower bounds can be correctly defined based on the extrema at specific points within the supporting stencil and thereafter the numerical oscillations in the adjacent region to the discontinuities can be suppressed effectively. The main advantage of this limiting technique lies in its simple methodology and easy implementation. Furthermore, the multi-dimensional character and the point extrema-based methodology endow the proposed limiter with ability to solve multi-dimensional flow problems on unstructured grids. A series of smooth and non-smooth numerical examples in one, two and three dimensions on unstructured grids are tested. Numerical results obtained demonstrate the superior performance of the proposed limiter on simulating compressible inviscid flows.

AMS subject classifications: 65M08, 76M12

Key words: Shock-capturing limiter, high-order finite volume method, least square-based finite difference, unstructured grids, compressible inviscid flows.

1 Introduction

High-order numerical methods enjoy high popularity in CFD due to their higher accuracy than low-order methods and they have been applied to various problems. Among

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them, the methods for unstructured grids are more active due to their excellent adaptability and flexibility to complex geometries. Various unstructured high-order finite volume (FV) methods have been developed in the past two decades. One pioneering example is the k -exact method [1–4]. Other typical ones include the moving least-squares (MLS) method [5,6], weighted essentially non-oscillatory (WENO) based methods [7–9], discontinuous Galerkin (DG) based hybrid method [10], compact least-squares (CLS) method [11], variational reconstruction method [12], and high-order gas-kinetic scheme (GKS) [13], etc. Recently, a high-order least square-based finite difference-finite volume (LSFD-FV) method on unstructured grids was proposed by Shu and his coworkers [14,15]. The LSFD-FV method applies a Taylor series expansion within each control cell as the solution approximation function. The unknown spatial derivatives are approximated by the mesh-free least square-based finite difference (LSFD) scheme using the functional values at centers of the current cell and its neighboring cells. As shown in the work [14], the high-order LSFD-FV method achieves superior accuracy and computational efficiency compared with the k -exact method for incompressible flows.

Although the aforementioned methods make a great contribution in solving several challenging or bottleneck problems on unstructured grids, the lack of mature shock-capturing techniques still limits their practical applications. In fact, either the second-order methods or the high-order methods assume that the linear or high-order distribution of flow variables on one supporting stencil is entirely continuous. However, many flow problems would involve discontinuities in the flow field, e.g., contact discontinuities and shock waves. The reconstruction or solution approximation would cause numerical oscillations near the discontinuities, i.e., the Gibbs phenomenon [16,17]. As a result, serious instability or even divergence may arise. Thus, both the second- and high-order methods require shock-capturing techniques to suppress numerical oscillations in the vicinities of the discontinuities. As a matter of fact, various techniques have been developed for the unstructured second-order methods, whereas the high-order methods starve for such techniques to suppress the more severe oscillations and maintain the designed accuracy in the smooth regions.

The existing shock-capturing techniques for unstructured high-order methods include the posteriori and priori ones. Multi-dimensional Optimal Order Detection (MOOD) [18–21], as the representative posteriori limiting strategy, was originally proposed to deal with the limitation procedure in the high-order FV methods. MOOD detects problematic situations based on several criteria after every evaluation and updates the candidate solution with the optimal low-order accurate one which satisfies the specific criteria. In the literature, the priori methods seem to be more common and they can be generally classified into three categories, i.e., the artificial viscosities [22–24], ENO/WENO schemes [7,8,25–27] and slope limiters [28–35]. The first technique adds some dissipation related to the flows to the governing equations. The merit of artificial viscosities is the easy implementation and excellent stability, while the defect is that the artificial viscosity has no physical meaning and needs to be adjusted according to the flow problems. Insufficient artificial viscosity cannot suppress numerical oscillations. Over-