Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **13**, No. 5, pp. 1027-1063

DOI: 10.4208/aamm.OA-2020-0388
October 2021

A Discontinuous Galerkin Method with Minimal Dissipation for a Finite-Strain Plate

Qiao Kang and Yan Xu*

School of Mathematical Sciences, University of Science and Technology of China, Hefei, Anhui 230026, China

Received 21 December 2020; Accepted (in revised version) 9 March 2021

Abstract. In this paper, we develop and analyze a discontinuous Galerkin (DG) method with minimal dissipation for the static bending problem of a finite-strain plate equation. The equations are deduced from a three-dimensional field equation. So the coupling of the equations and the mixed derivative terms are the barriers during developing discretization schemes. The error estimates of the scheme are proved in detail. Numerical experiments in different circumstances are presented to demonstrate the capabilities of the method.

AMS subject classifications: 65N30, 35J47, 35G15

Key words: Finite-strain, static bending problem, discontinuous Galerkin methods, numerical fluxes, error estimates.

1 Introduction

In this paper, we present a discontinuous Galerkin (DG) method for a finite-strain plate model [20]:

$$\frac{1}{2}\bar{E}[(1-\nu)\Delta u+(1+\nu)(u_x+v_y)_x]-\bar{E}\hbar\Delta w_x+f_1=0, \tag{1.1a}$$

$$\frac{1}{2}\bar{E}[(1-\nu)\Delta v+(1+\nu)(u_x+v_y)_y]-\bar{E}\hbar\Delta w_y+f_2=0, \tag{1.1b}$$

$$-\bar{E}\hbar\Delta(u_x+v_y)+\frac{2}{3}\bar{E}\hbar^2\Delta^2 w+f_3=0, \tag{1.1c}$$

with boundary condition

$$u\big|_{\partial\Omega}=g_1, \qquad v\big|_{\partial\Omega}=g_2, \tag{1.2a}$$

$$w\big|_{\partial\Omega}=g_3, \qquad \nabla w\cdot\boldsymbol{n}\big|_{\partial\Omega}=g_4, \tag{1.2b}$$

*Corresponding author.
Emails: kq1994@mail.ustc.edu.cn (Q. Kang), yxu@ustc.edu.cn (Y. Xu)

where $2\hbar$ represents the thickness of the plate, $\bar{E}=E/(1-\nu^2)$ with Young's modulus E and ν denotes the Poisson's ratio.

This model describes the steady state of a finite-strain isotropic plate with compression and limitation. Different from traditional plate equation, this model consider the plate with a certain thickness. Due to the definition of Poisson's ratio, we know that if a sample of material is stretched in one direction, it tends to get thinner in the other two directions. It reminds us to consider the offset along plate surface when the plate vibrates. In this case, Song Zilong and Dai Hui-Hui starts from the three-dimensional field equations for a compressible hyperelastic material, by a series expansion about the bottom surface, they deduce a vector dynamic plate equation with three unknowns, then eliminate all the time dependent terms and get a static bending problem, which is this model.

Although there are abundant models for thin plates, the model in (1.1) is different. Comparing with the classical theory [22], the system (1.1) has the same order with similar four boundary conditions, and the equation for the transverse displacement w can be decoupled from the other two for the isotropic static case.

$$\frac{1}{2}[(1-\nu)\Delta u+(1+\nu)(u_x+v_y)_x]-\hbar\Delta w_x=F_1, \tag{1.3a}$$

$$\frac{1}{2}[(1-\nu)\Delta v+(1+\nu)(u_x+v_y)_y]-\hbar\Delta w_y=F_2, \tag{1.3b}$$

$$\Delta^2 w=F_3. \tag{1.3c}$$

Moreover, the present model produces more accurate results for deflection in this static bending problem. The another model in [16] produces significant accurate deflection in the static bending problem, but it's more complex. It involves five unknowns and needs five boundary conditions at each edge point. Further more, we know that Reddy's plate theory [17] is a good theory, it inherits the merits of the theory in [16] and avoids the need for shear correction factor. It also successfully captures the quadratic distribution of shear stresses in the bending mode as the present one, but at the expense of its complexity. Specifically, it involves five unknowns and needs six boundary conditions (on the edge) with also an indispensable higher-order stress resultant. In contrast, the present theory entails only well-known physical quantities on the boundary. Also, the present theory produces more accurate distributions for all concerned quantities, and is expected to apply to more general loadings.

In this paper, we will develop and analysis discontinuous Galerkin (DG) methods for the system (1.3). The DG methods for high order spatial derivatives have been intensively investigated, such as local discontinuous Galerkin (LDG) methods [10], Baumann-Oden DG methods [1], direct discontinuous Galerkin methods [14, 15], inner penalty Galerkin methods [3, 4, 7, 13, 19, 21, 33], conforming DG methods [29–31], etc. In this paper, the DG framework is mainly enlightened by the work [8], in which Cockburn and Dong Bo constructed a LDG scheme for convection-diffusion problem.