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Algebraic Multigrid Block Triangular Preconditioning for Multidimensional Three-Temperature Radiation Diffusion Equations

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Abstract. In the paper, we are interested in block triangular preconditioning techniques based on algebraic multigrid approach for the large-scale, ill-conditioned and 3-by-3 block-structured systems of linear equations originating from multidimensional three-temperature radiation diffusion equations, discretized in space with the symmetry-preserving finite volume element scheme. Both lower and upper block triangular preconditioners are developed, analyzed theoretically, implemented via the two-level parallelization and tested numerically for such linear systems to demonstrate that they lead to mesh-independent convergence behavior and scale well both algorithmically and in parallel.

AMS subject classifications: 65F10, 65F15, 65N55

Key words: Radiation diffusion equations, algebraic multigrid, block triangular preconditioning, parallel computing.

1 Introduction

Radiation transport is one of the fundamental physical processes in the laser indirect-drive inertial confinement fusion (ICF). Due to the high nonlinearity in optically thick

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medium, simulating radiation transport is practically very difficult. In the current context, radiation transport process is calculated using a flux-limited three-temperature (3-T) radiation diffusion approximation [5, 28, 35]. However, because of the complicated nonlinear couplings among their physical quantities at multiple temporal and spatial scales, 3-T radiation diffusion equations are commonly discretized by the finite volume method allowing for local conservations [14,26,29,44,52,54,56,62], resulting in a sequence of large, sparse and (non)symmetric but positive definite linear systems. It should be stressed that obtaining these approximate solutions is challenging, since the corresponding matrices are quite ill-conditioned, accounting for more than 80% of the total ICF simulation run time. The demand for more accurate ICF simulations has led to larger linear systems that are more difficult, or even impossible, to be solved in an acceptable amount of time using standard direct or conventional iterative solvers. Robust and scalable solvers that do not deteriorate as the mesh resolution is refined are therefore needed to harness the computational power of current supercomputers.

In the literature, there are a variety of different preconditioned Krylov subspace methods [6, 30, 31, 39, 64, 66, 67, 71–73]. It should be mentioned that these preconditioners mainly fall into several preconditioning algorithms, e.g., the incomplete LU factorization (ILU), monolithic algebraic multigrid (AMG), domain decomposition (DD) preconditioners together with their symmetric and nonsymmetric combinations. Since each of those coefficient matrices has an underlying block structure, one can also construct certain natural block preconditioners to separate the global problem into easier-to-solve subproblems, form an object-oriented framework to incorporate existing single-physics solvers/preconditioning strategies within the framework of a monolithic solver. Block preconditioners with AMG applied as a key component have been proven very successful in numerous applications, e.g., poroelastic equations [1], Maxwell's equations [2], liquid crystal directors model [7], geophysical electromagnetics [8], multiphase poromechanics of heterogeneous media [11], linear elasticity in mixed form [12], Stokes problem [13], fluid-structure interaction problems in hemodynamics [17, 36], incompressible Navier-Stokes problem [19], fault/fracture mechanics saddle-point problems [25], elliptic optimal control problems [27], saddle point problems with a penalty term [33], multiphase reservoir flow and geomechanics [34], incompressible magnetohydrodynamics models [38, 45, 60], PDE-constrained optimization problems [42, 46], models of coupled magma/mantle dynamics [47], Brinkman problem [58] and all-speed melt pool flow physics [61].

For multidimensional radiation diffusion equations, Mousseau, Knoll and Rider [41] and An et al. [3,23,24] proposed various operator-based preconditioners in Jacobian-free Newton-Krylov framework; Brown and Woodward [10] considered the Schur complement preconditioner to precondition the generalized minimum residual (GMRES) solver without restarting, and demonstrated a fully implicit solver based on some higher-order time integration; the physical-variable based coarsening two-level preconditioner was proposed by Xu, Mo and An [65] and further improved by Shu et al. [75,77]; two types of block Schur complement preconditioners conceived as approximate inverses of the coeffi-