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Explicit High Order One-Step Methods for Decoupled Forward Backward Stochastic Differential Equations

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Abstract. By using the Feynman-Kac formula and combining with Itô-Taylor expansion and finite difference approximation, we first develop an explicit third order one-step method for solving decoupled forward backward stochastic differential equations. Then based on the third order one, an explicit fourth order method is further proposed. Several numerical tests are also presented to illustrate the stability and high order accuracy of the proposed methods.

AMS subject classifications: 65C20, 65C30, 60H35, 65H30

Key words: Decoupled forward backward stochastic differential equations, Itô-Taylor expansion, finite difference approximation, explicit one-step method, high order convergence.

1 Introduction

Let $T>0$ be a deterministic terminal time and $(\Omega,\mathcal{F},\mathbb{F},P)$ denote a filtered complete probability space with the natural filtration $\mathbb{F}=(\mathcal{F}_t)_{0\leq t\leq T}$ of an m-dimensional Brownian motion $W=(W_t)_{0\leq t\leq T}$. Consider the decoupled forward backward stochastic differential equations (FBSDEs) on $(\Omega,\mathcal{F},\mathbb{F},P)$

$$\begin{cases} X_t=X_0+\int_0^t b(s,X_s)ds+\int_0^t \sigma(s,X_s)dW_s, \\ Y_t=\varphi(X_T)+\int_t^T f(s,X_s,Y_s,Z_s)ds-\int_t^T Z_s dW_s, \end{cases} \tag{1.1}$$

where $t\in[0,T]$, $X_0\in\mathcal{F}_0$ is an initial condition; $b:[0,T]\times\mathbb{R}^d\to\mathbb{R}^d$ and $\sigma:[0,T]\times\mathbb{R}^d\to\mathbb{R}^{d\times m}$ are, respectively, the drift and diffusion coefficients of stochastic differential equations

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(SDEs); $\varphi:\mathbb{R}^d\to\mathbb{R}^n$ is the terminal condition and $f:[0,T]\times\mathbb{R}^d\times\mathbb{R}^n\times\mathbb{R}^{n\times m}\to\mathbb{R}^n$ is the generator of backward stochastic differential equations (BSDEs). A triple (X_t,Y_t,Z_t) is called an L^2-adapted solution of the FBSDE (1.1) if it is $\mathcal{F}_t$-adapted, square integrable and satisfies (1.1).

Pardoux and Peng [22] first proved the existence and uniqueness of the adapted solution of the nonlinear BSDEs. Then by using the solutions of BSDEs, Peng [24] gave a probabilistic interpretation for quasilinear parabolic partial differential equations (PDEs). Since then, the study on FBSDEs has been extensively conducted due to its applications in research on PDEs [6, 17, 24], mathematical finance [11, 15, 20], stochastic optimal control [14, 23], and mean-field BSDEs [2, 3, 26, 28], to name a few. However, the analytic solutions of FBSDEs are seldom known. Hence, it is important and popular to solve FBSDEs with some numerical methods.

Up to now, there have been a considerable number of numerical methods for solving BSDEs [4,5,12,13,31–33,36,37] and decoupled FBSDEs [7–10,18,19,25,29,30,34,35,38,39] in literatures. Nevertheless, except some of the multistep methods [30, 34, 37, 39], few of one-step methods can achieve high order convergence exceeding two. In [31], the authors constructed a class of third-order one-step methods for solving BSDEs whose driver f is independent of Z. This is the first attempt to study the third order one-step method for solving BSDEs. However, this one-step method can only solve BSDEs with f not taking Z as input, which causes significant limitations in application.

In this paper, we aim to design two high order one-step methods for solving general decoupled FBSDEs by extending the method given in [31]. Based on the Feynman-Kac formula, by combining with the Itô-Taylor expansions and the high order finite difference approximations, we first develop an explicit third order one-step method containing a parameter θ for FBSDEs. Then by taking $\theta=\frac{1}{2}$ and utilizing the prediction-correction method, we further propose a fourth order one-step method for FBSDEs. To attest the stability and high order accuracy of the proposed methods, we carry out some numerical experiments. All of the numerical results show that both methods are stable and high order accurate, and the fourth order method can achieve a fourth order accuracy when the weak order 4.0 Itô-Taylor scheme is used to solve SDEs.

The rest of the paper is organized as follows. In Section 2, some preliminaries on the Feynman-Kac formula, Itô-Taylor expansion, and finite difference approximation are presented, then based on which we propose an explicit third order one-step method and an explicit fourth order one-step method for decoupled FBSDEs in Section 3. In Section 4, some numerical tests are presented to show the stability and high order accuracy of the proposed methods. Finally, we give some conclusions in Section 5.

2 Preliminaries

2.1 The Feynman-Kac formula