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Integrating Krylov Deferred Correction and Generalized Finite Difference Methods for Dynamic Simulations of Wave Propagation Phenomena in Long-Time Intervals

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Abstract. In this paper, a high-accuracy numerical scheme is developed for long-time dynamic simulations of 2D and 3D wave propagation phenomena. In the derivation of the present approach, the second order time derivative of the physical quantity in the wave equation is treated as a substitution variable. Based on the temporal discretization with the Krylov deferred correction (KDC) technique, the original wave problem is then converted into the modified Helmholtz equation. The transformed boundary value problem (BVP) in space is efficiently simulated by using the meshless generalized finite difference method (GFDM) with Taylor series after truncating the second and fourth order approximations. The developed scheme is finally verified by four numerical experiments including cases with complicated domains or the temporally piecewise defined source function. Numerical results match with the analytical solutions and results by the COMSOL software, which demonstrates that the developed KDC-GFDM can allow large time-step sizes for wave propagation problems in long-time intervals.

AMS subject classifications: 35L05, 65M06

Key words: Wave equation, Krylov deferred correction technique, large time-step, long-time simulation, generalized finite difference method.

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1 Introduction

The wave equation is a very momentous partial differential equation (PDE) involved in

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many physical applications, such as vibrations in strings and propagations of acoustic waves and water waves. The development of accurate and efficient numerical methods for simulations of wave propagation phenomena is an active research field [1–5]. Compared with mesh-dependent numerical algorithms, meshless methods, such as the method of fundamental solutions (MFS) [6–8], the method of approximate particular solutions (MAPS) [9,10], the boundary element-free method (BEFM) [11,12], and the singular boundary method (SBM) [13–19], have been one kind of powerful techniques because of no requirement of laborious mesh generation especially for 3D problems with complicated geometries.

Various meshless methods have so far been applied to the simulation of wave equation [20–23]. Dehghan and Shokri [20] introduced the radial basis functions (RBF) method to solve the one-dimensional (1D) wave equation with coupling θ-weighted scheme. Zhang et al. [21] proposed an improved element-free Galerkin method (EFGM) for the damped wave propagation and discretized the time by the second-order center finite difference scheme. Based on the same technique of time discretization, Wang et al. [22] developed a central compact finite difference scheme for acoustic wave problems. Ureña et al. [23] applied the generalized finite difference method (GFDM) to the two-dimensional (2D) seismic wave propagation problem. The GFDM, as a very powerful meshless method, has been widely used for many applications [24–31] because of its simplicity, stability and easy implementation.

For time dependent wave problems, the finite difference scheme is a common choice for the discretization in temporal direction, such as those in [20–23]. However, the time step size has to be limited due to stability issues even for the five-stage Runge–Kutta method [32]. In long-time dynamic simulations, larger time step size is one of key factors to reduce the accumulation of the temporal-error. Several approaches were developed to allow larger time step size for temporal discretization, such as the spectral deferred correction (SDC) method [33, 34], the integral deferred correction (InDC) method [35], and the Krylov deferred correction (KDC) method [36–39]. The KDC method applies the SDC as a preconditioner for the collocation formulations and then uses the Newton-Krylov technique to the solution of resulting equations, which reduces the number of iterations to converge.

By integrating advantages of the KDC technique and the meshless GFDM, a high-accuracy numerical approach called as the KDC-GFDM is constructed in this paper for dynamic simulations of 2D and 3D long-time wave propagation phenomena. For the developed algorithm, we consider the second order time derivative of the physical quantity in the wave equation as a substitution variable. Based on the temporal discretization with the KDC approach, the wave equation is then changed into the modified Helmholtz equation. The solution of the transformed boundary value problem (BVP) is determined by using the GFDM. The outline of this paper is as follows. The details of the developed algorithm are given in Section 2. 2D and 3D numerical examples including cases with the temporally piecewise defined source function or complicated domains are provided in Section 3. Some discussions are concluded in Section 4.