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WENO Interpolation-Based and Upwind-Biased Free-Stream Preserving Nonlinear Schemes

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Abstract. For flow simulations with complex geometries and structured grids, it is preferred for high-order difference schemes to achieve high accuracy. In order to achieve this goal, the satisfaction of free-stream preservation is necessary to reduce the numerical error in the numerical evaluation of grid metrics. For the linear upwind schemes with flux splitting the free-stream preserving property has been achieved in the early study [Q. Li et al., Commun. Comput. Phys., 22 (2017), pp. 64–94]. In the current paper, new series of nonlinear upwind-biased schemes through WENO interpolation will be proposed. In the new nonlinear schemes, the shock-capturing capability on distorted grids is achieved, which is unavailable for the aforementioned linear upwind schemes. By the inclusion of fluxes on the midpoints, the nonlinearity in the scheme is obtained through WENO interpolations, and the upwind-biased construction is acquired by choosing relevant grid stencils. New third- and fifth-order nonlinear schemes are developed and tested. Discussions are made among proposed schemes, alternative formulations of WENO and hybrid WCNS, in which a general formulation of center scheme with midpoint and nodes employed is obtained as a byproduct. Through the numerical tests, the proposed schemes can achieve the designed orders of accuracy and free-stream preservation property. In 1-D Sod and Shu-Osher problems, the results are consistent with the theoretical predictions. In 2-D cases, the vortex preservation, supersonic inviscid flow around cylinder at $M_\infty = 4$, Riemann problem, and shock-vortex interaction problems have been tested. More specifically, two types of grids are employed, i.e., the uniform/smooth grids and the randomized/locally-randomized grids. All schemes can get satisfactory results in uniform/smooth grids. On the randomized grids, most schemes have accomplished computations with reasonable accuracy, except the failure of one third-order scheme in Riemann problem and shock-vortex interaction. Overall, the proposed nonlinear schemes have the capability to solve flow problems on badly deformed grids, and the schemes can be used in the engineering applications.

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1 Introduction

On developing high-order difference schemes, large advances have been achieved since the presence of WENO scheme series [1–13]. Besides WENO schemes, high-order compact schemes [14] were proposed as well with well-designed dispersion relations. Despite of abundant researches and outcomes, it is interesting to observe that high-order difference schemes are seldom applied for engineering complexities such as airplanes. In such situations, multi-blocked grids are inevitably used, the variation of grid topology usually happens, and meshes with locally bad quality might be encountered. For example, distorted grids would exist either inside blocks or on boundaries between them. Regarding the former one, Visbal and Gaitonde [15] showed that large errors may arise in computations, and similar conclusions were made in the subsequent studies [16,24]. Regarding the latter one, it was reported [18] that spurious oscillations would occur around the grid conjunctions. It is known that one of the causes would be the mismatches between evaluating grid metrics and solving conservative governing equations in curvilinear coordinate system.

In deriving conservative governing equations, theoretically zero-valued identities, namely metric identities, are introduced in partial differential operations [19]. However, the identities might not be null in practical computations [20]. In order to minimize such metric-caused errors, the following understandings are reached: (1) The grid metrics are suggested to adopt the conservative form rather than the expanded one [21,22]; (2) The consistent use of difference scheme in each coordinate direction should be applied to discretize the spatial derivatives in grid metrics and governing equations [15,23]. The key issue involved is the commutativity of difference operators [23], which was proved in [23] first and [24] later. With the help of commutativity, for an initially uniform flow, the flow will remain constant all the time, i.e., the satisfaction of the so-called free-stream preservation (FSP) property. Although FSP may be achieved by either linear central or upwind schemes, nonlinear mechanisms should become necessary to ensure numerical stability for the flow with possible discontinuity. In this regard, two methodologies have been employed for linear central schemes, i.e., filtering [15] and WENO interpolations [17,18]. For linear upwind schemes, however, the consistent use of certain scheme might invoke numerical instability because flow characteristics may contain both upwind and downwind components. Moreover, the flux is usually split into two parts with upwind discretization, the conventional implementation may make the schemes not to meet the requirement of the FSP property.