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Localized Method of Fundamental Solutions for Three-Dimensional Elasticity Problems: Theory

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Abstract. A localized version of the method of fundamental solution (LMFS) is devised in this paper for the numerical solutions of three-dimensional (3D) elasticity problems. The present method combines the advantages of high computational efficiency of localized discretization schemes and the pseudo-spectral convergence rate of the classical MFS formulation. Such a combination will be an important improvement to the classical MFS for complicated and large-scale engineering simulations. Numerical examples with up to 100,000 unknowns can be solved without any difficulty on a personal computer using the developed methodologies. The advantages, disadvantages and potential applications of the proposed method, as compared with the classical MFS and boundary element method (BEM), are discussed.

AMS subject classifications: 62P30, 65M32, 65K05

Key words: Method of fundamental solutions, meshless method, large-scale simulations, elasticity problems.

1 Introduction

The method of fundamental solutions (MFS) has emerged as a robust boundary-type meshless method for the solutions of certain boundary value problems [1–8]. The method won the favor of many researchers in engineering and science due to its advantage of high accuracy for many engineering applications [7, 9–13]. The classical MFS approach,

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however, produces dense and non-symmetric matrix of algebraic equations that requires memory and other operators to compute the unknown coefficients [14–18]. This makes the method limited to solving small-scale problems with thousands of degrees of freedom for a long time [7,19–24].

To overcome the aforementioned bottleneck associated with the classical MFS, a localized version of the method, named as the localized MFS (LMFS), is proposed by Fan and his coworkers [25–27]. In the LMFS approach, the whole computational domain is divided into a set of overlapping local subdomains in which the classical MFS approximation and moving least square (MLS) techniques are employed. Since the final system of algebraic equations is sparse, the computational efficiency of the method has been fully improved and the method can now be easily used to simulate large-scale applied mechanics problems. This paper documents the first attempt to apply the method for the numerical solutions of 3D elasticity problems. Some possible improvement as well as the influence of several factors on the overall accuracy of the method are also discussed. Numerical examples with up to 100,000 unknowns are solved successfully on a Core (TM) i7 PC using the developed LMFS code. A self-contained Matlab code is provided in the end of the paper.

2 Statement of the basic problem

The well-known Cauchy-Navier equations for 3D elasticity problems are [28–30]:

$$\left(\frac{2-2\nu}{1-2\nu}\right)\frac{\partial^2 u_1}{\partial x_1^2}+\frac{\partial^2 u_1}{\partial x_2^2}+\frac{\partial^2 u_1}{\partial x_3^2}+\left(\frac{1}{1-2\nu}\right)\frac{\partial^2 u_2}{\partial x_1\partial x_2}+\left(\frac{1}{1-2\nu}\right)\frac{\partial^2 u_3}{\partial x_1\partial x_3}=0, \tag{2.1a}$$

$$\left(\frac{1}{1-2\nu}\right)\frac{\partial^2 u_1}{\partial x_1\partial x_2}+\frac{\partial^2 u_2}{\partial x_1^2}+\left(\frac{2-2\nu}{1-2\nu}\right)\frac{\partial^2 u_2}{\partial x_2^2}+\frac{\partial^2 u_2}{\partial x_3^2}+\left(\frac{1}{1-2\nu}\right)\frac{\partial^2 u_3}{\partial x_2\partial x_3}=0, \tag{2.1b}$$

$$\left(\frac{1}{1-2\nu}\right)\frac{\partial^2 u_1}{\partial x_1\partial x_3}+\left(\frac{1}{1-2\nu}\right)\frac{\partial^2 u_2}{\partial x_2\partial x_3}+\frac{\partial^2 u_3}{\partial x_1^2}+\frac{\partial^2 u_3}{\partial x_2^2}+\left(\frac{2-2\nu}{1-2\nu}\right)\frac{\partial^2 u_3}{\partial x_3^2}=0, \tag{2.1c}$$

with the following displacement and/or traction boundary conditions:

$$u_i=\bar{u}_i \qquad \text{on boundary } \Gamma_u, \tag{2.2a}$$

$$t_i=\bar{t}_i \qquad \text{on boundary } \Gamma_t, \tag{2.2b}$$

where u_i and t_i denote displacements and boundary tractions, respectively, the barred quantities $\bar{u}_i$ and $\bar{t}_i$ represent known boundary conditions, and ν stands for the Poisson's ratio. According to theory of linear elasticity, the strains (ε_{ij}) and stresses (σ_{ij}) are related to displacements as

$$\varepsilon_{ij}=\frac{1}{2}\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right), \tag{2.3a}$$

$$\sigma_{ij}=2\mu\left(\varepsilon_{ij}+\frac{\nu}{1-2\nu}\varepsilon_{kk}\delta_{ij}\right), \tag{2.3b}$$